Spherical Microphone Arrays

Acoustic Wave Equation

- Helmholtz Equation
  - Assuming the solutions of wave equation

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \nabla^2 \psi$$

are time harmonic waves of frequency ω

$$\psi(\mathbf{r}, t) = e^{-j\omega t} \phi(\mathbf{r})$$

  - $\phi(\mathbf{r})$ satisfies the *homogeneous Helmholtz equation*:

$$(\nabla^2 + k^2)\phi(\mathbf{r}) = 0$$

Boundary Value Problems

- Dirichlet:

$$\psi|_S = 0,$$

- Neumann:

$$\left.\frac{\partial \psi}{\partial n}\right|_S = 0,$$

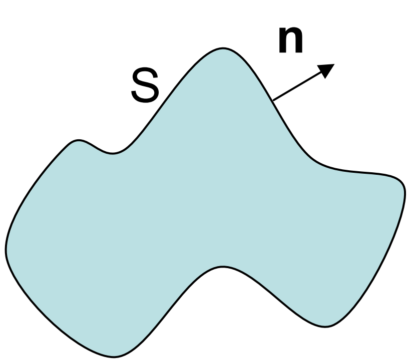
- Robin:

$$\left.\left(\frac{\partial \psi}{\partial n} + i\sigma\psi\right)\right|_S = 0.$$

- Sommerfield Radiation Condition (for external problems):

$$\psi = \psi_{in} + \psi_{scat}$$

$$\lim_{r\to\infty}\left[r\left(\frac{\partial \psi_{scat}}{\partial r} - ik\psi_{scat}\right)\right] = 0.$$



Green's Function and Identity

Free space Green's function:

$$\nabla^2 G(\mathbf{x},\mathbf{y}) + k^2 G(\mathbf{x},\mathbf{y}) = -\delta(\mathbf{x}-\mathbf{y}),$$

$$G(\mathbf{x},\mathbf{y}) = \frac{\exp(ik|\mathbf{x}-\mathbf{y}|)}{4\pi|\mathbf{x}-\mathbf{y}|}, \quad \mathbf{x},\mathbf{y} \in \mathbb{R}^3.$$

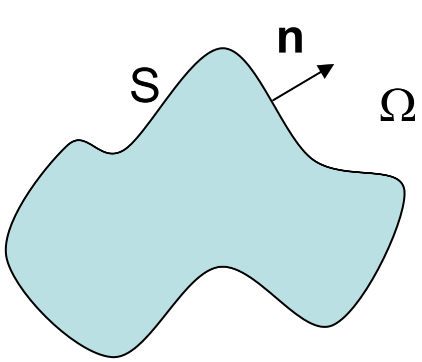
Green's formula:

$$\psi(\mathbf{y}) = \int_S \left[\psi(\mathbf{x}) \frac{\partial G(\mathbf{x},\mathbf{y})}{\partial n(\mathbf{x})} - G(\mathbf{x},\mathbf{y}) \frac{\partial \psi(\mathbf{x})}{\partial n(\mathbf{x})} \right] \mathrm{d}S(\mathbf{x}), \quad \mathbf{y} \in \Omega.$$

Boundary integral equation

$$\alpha\psi(\mathbf{y}) = \int_S \left(\psi(\mathbf{x}) \frac{\partial G(\mathbf{x},\mathbf{y})}{\partial n(\mathbf{x})} - G(\mathbf{x},\mathbf{y}) \frac{\partial \psi(\mathbf{x})}{\partial n(\mathbf{x})} \right) \mathrm{d}S(\mathbf{x}),$$

$$\alpha = \begin{cases} \frac{1}{2} & \mathbf{y} \text{ on a smooth part of the boundary} \\ \frac{\gamma}{4\pi} & \mathbf{y} \text{ at a corner on the boundary} \\ 1 & \mathbf{y} \text{ inside the domain} \end{cases}.$$



Distributions of Monopoles and Dipoles

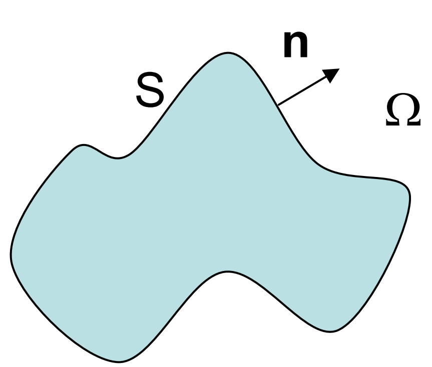
Volume source distribution:

$$\psi(\mathbf{y}) = \sum_{j=1}^{N} Q_j G(\mathbf{x}_j, \mathbf{y}), \quad \mathbf{y} \in \mathbb{R}^3 \backslash \{\mathbf{x}_j\},$$

$$\psi(\mathbf{y}) = \int_{\overline{\Omega}} q(\mathbf{x}) G(\mathbf{x}, \mathbf{y}) \mathrm{d}V(\mathbf{x}), \quad \mathbf{y} \in \Omega, \quad \overline{\Omega} \cap \Omega = \emptyset.$$

Single layer potential:

$$\psi(\mathbf{y}) = \int_S q_\sigma(\mathbf{x}) G(\mathbf{x}, \mathbf{y}) \mathrm{d}S(\mathbf{x}), \quad \mathbf{y} \in \Omega, \quad S = \partial\Omega.$$



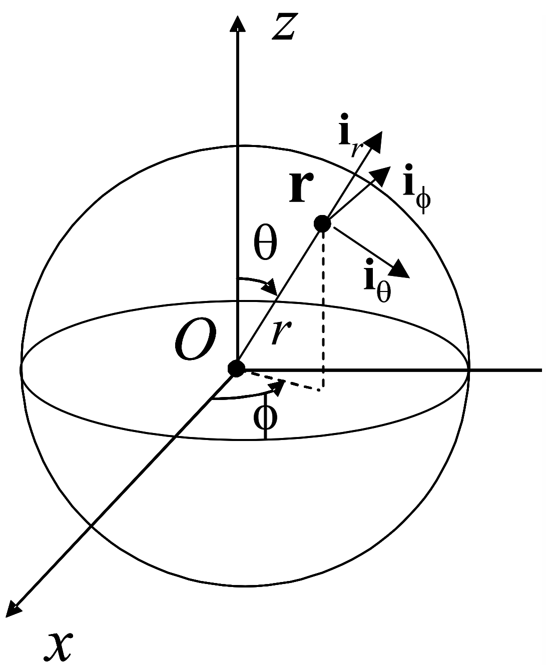
Double layer potential:

$$\psi(\mathbf{y}) = \int_S q_\mu(\mathbf{x}) \frac{\partial G(\mathbf{x}, \mathbf{y})}{\partial n(\mathbf{x})} \mathrm{d}S(\mathbf{x}), \quad \mathbf{y} \in \Omega, \quad S = \partial\Omega.$$

Expansions in Spherical Coordinates

Spherical Basis Functions

$$x = r\sin\theta\cos\varphi, \quad y = r\sin\theta\sin\varphi, \quad z = r\cos\theta.$$



Spherical Coordinates

Spherical Bessel Functions

Regular Basis Functions

$$R_n^m(\mathbf{r}) = j_n(kr)Y_n^m(\theta,\varphi),$$

Singular Basis Functions

$$S_n^m(\mathbf{r}) = h_n(kr)Y_n^m(\theta,\varphi).$$

Spherical Hankel Functions of the First Kind

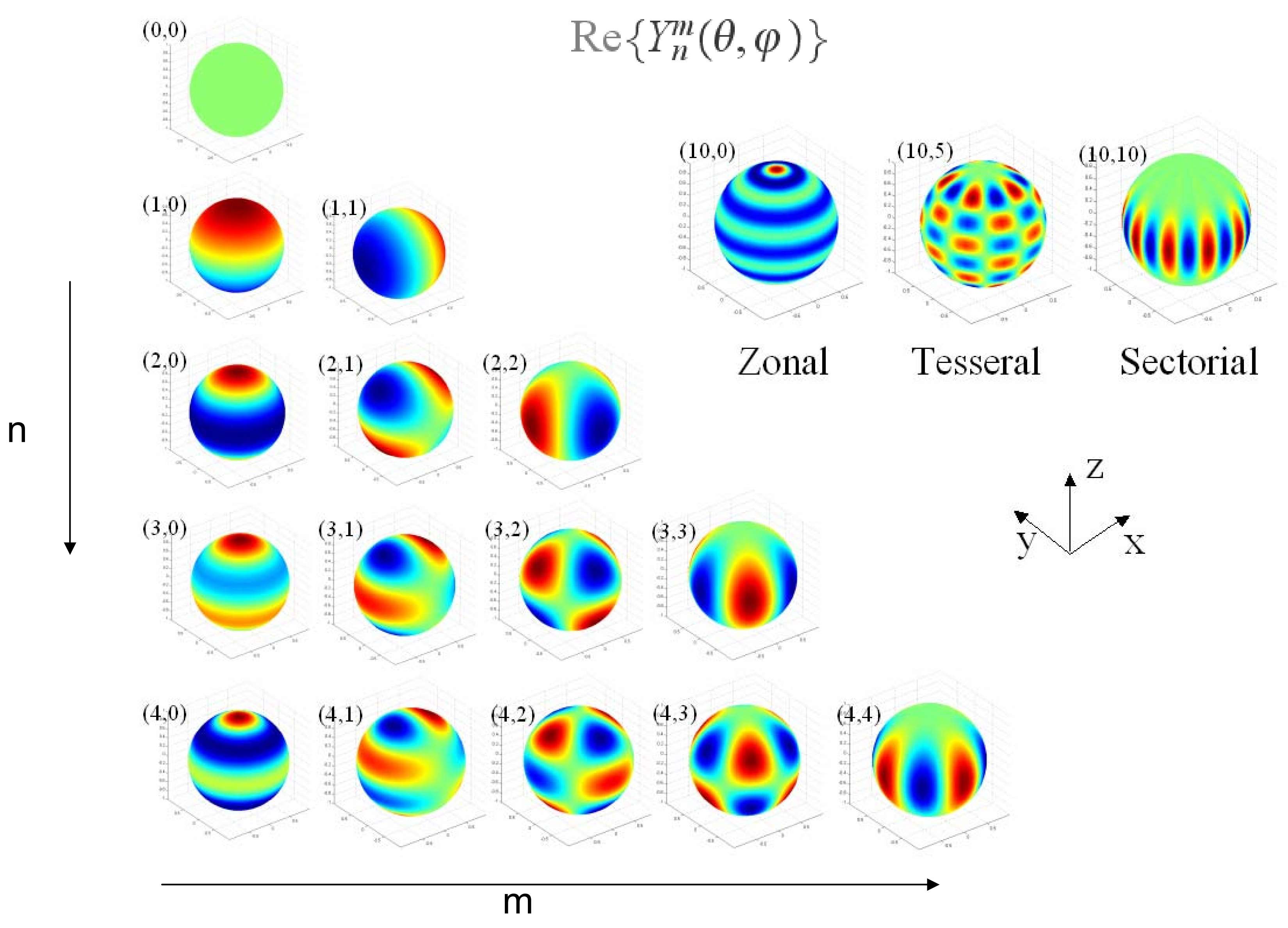
Spherical Harmonics

$$Y_n^m(\theta,\varphi) = (-1)^m \sqrt{\frac{2n+1}{4\pi}\frac{(n-|m|)!}{(n+|m|)!}}P_n^{|m|}(\cos\theta)e^{im\varphi},$$

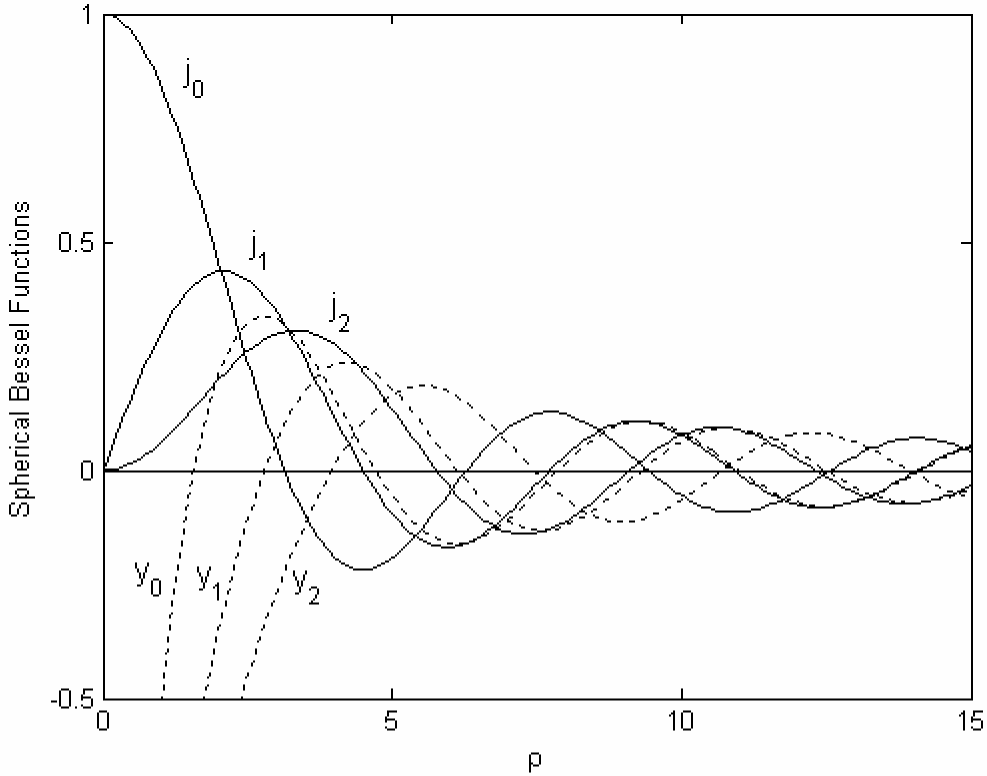
$$n = 0,1,2,\ldots; \qquad m = -n,\ldots,n.$$

Associated Legendre Functions

Spherical Harmonics



Spherical Bessel Functions



$$h_n(\rho) = j_n(\rho) + i y_n(\rho)$$

$$j_0(\rho) = \frac{\sin\rho}{\rho}, \quad j_1(\rho) = \frac{\sin\rho}{\rho^2} - \frac{\cos\rho}{\rho},$$

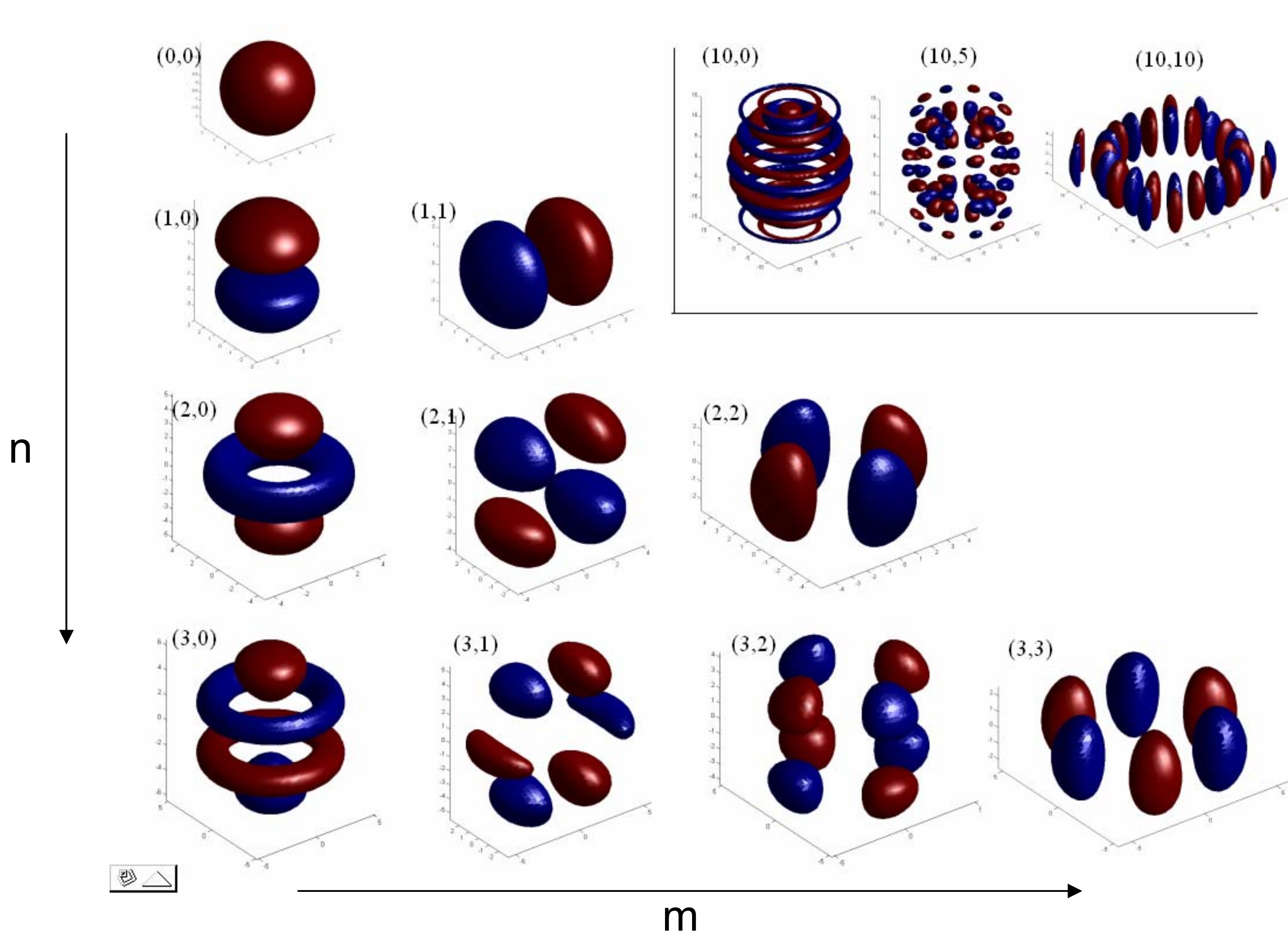
$$j_2(\rho) = \left(\frac{3}{\rho^3} - \frac{1}{\rho}\right)\sin\rho - \frac{3}{\rho^2}\cos\rho,$$

$$y_0(\rho) = -\frac{\cos\rho}{\rho}, \quad y_1(\rho) = -\frac{\cos\rho}{\rho^2} - \frac{\sin\rho}{\rho},$$

$$y_2(\rho) = \left(-\frac{3}{\rho^3} + \frac{1}{\rho}\right)\cos\rho - \frac{3}{\rho^2}\sin\rho.$$

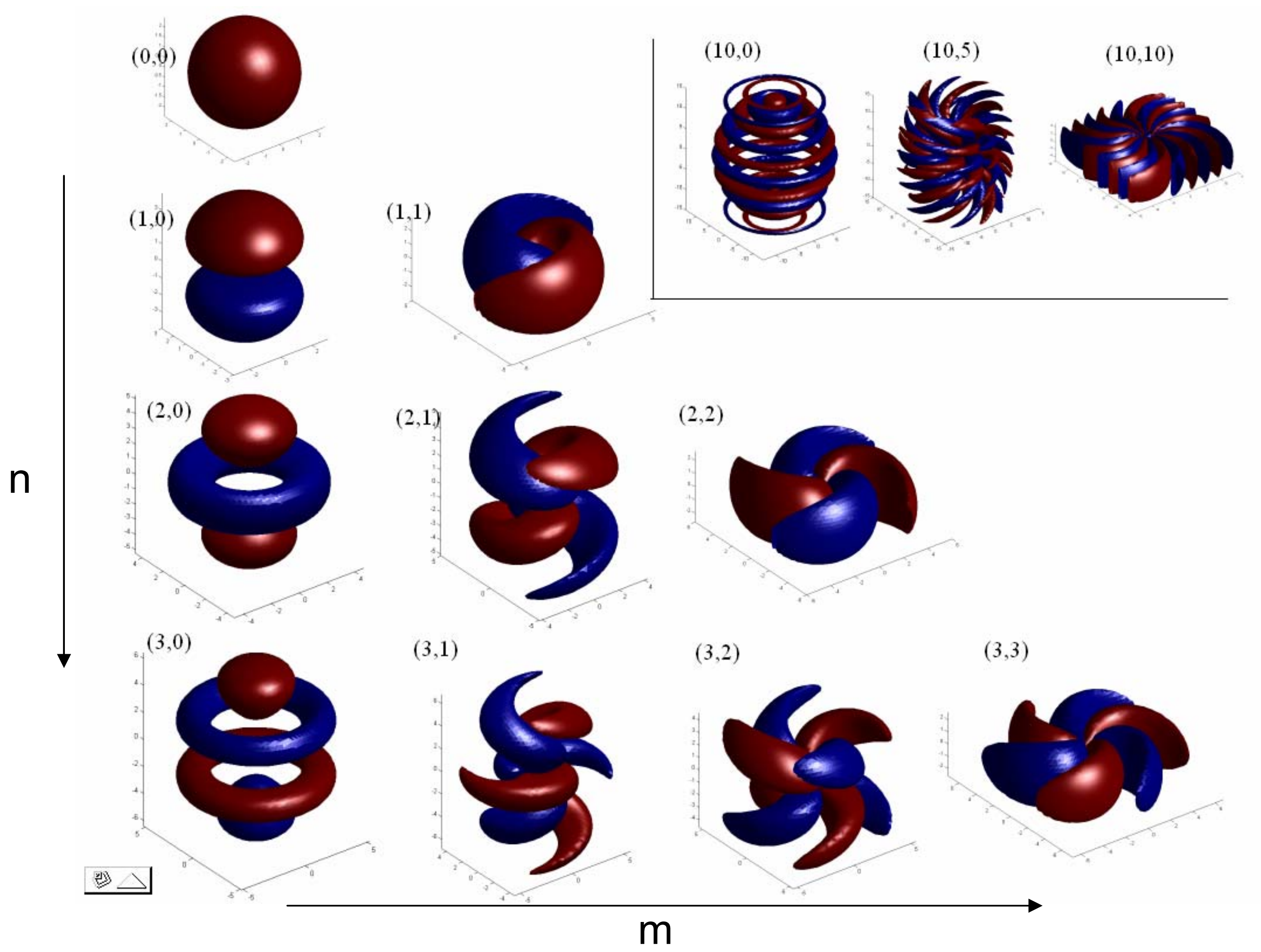
Isosurfaces For Regular Basis Functions

$$\mathrm{Re}\{R_n^m(\mathbf{r})\} = const$$



Isosurfaces For Singular Basis Functions

$$\mathrm{Re}\{S_n^m(\mathbf{r})\} = const$$



Expansions

$$\psi(\mathbf{r}) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} A_n^m F_n^m(\mathbf{r}) = \sum_{m=-\infty}^{\infty}\sum_{n=|m|}^{\infty} A_n^m F_n^m(\mathbf{r}), \quad F = S, R, \quad A_n^m \in \mathbb{C}.$$

Absolute and uniform convergence

$$\forall \epsilon > 0, \quad \exists p(\epsilon), \quad \left| \psi(\mathbf{r}) - \sum_{n=0}^{p-1}\sum_{m=-n}^{n} A_n^m F_n^m(\mathbf{r}) \right| < \epsilon, \quad \forall \mathbf{r} \in \Omega,$$

and

$$\forall \epsilon > 0, \quad \exists p(\epsilon), \quad \sum_{n=p}^{\infty}\sum_{m=-n}^{n} |A_n^m F_n^m(\mathbf{r})| < \epsilon, \quad \forall \mathbf{r} \in \Omega.$$

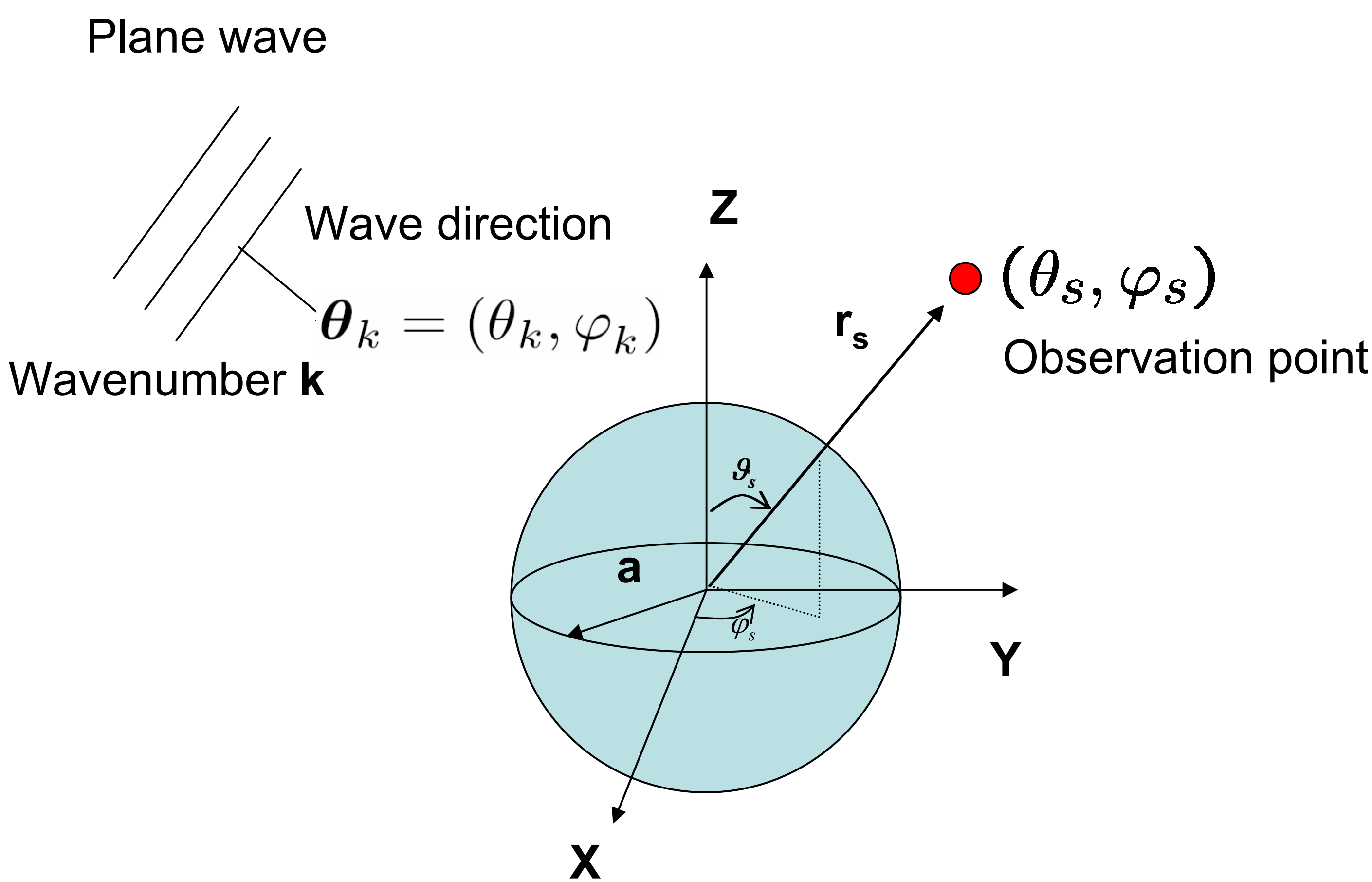
Plane Wave expansion:

$$e^{i\mathbf{k}\cdot\mathbf{r}} = 4\pi \sum_{n=0}^{\infty}\sum_{m=-n}^{n} i^n Y_n^{-m}(\theta_k, \varphi_k) R_n^m(\mathbf{r}),$$

$$\mathbf{k} = k\mathbf{s}, \quad \mathbf{s} = (\sin\theta_k \cos\varphi_k, \sin\theta_k \sin\varphi_k, \cos\theta_k).$$

Wave vector

Notations



Plane wave
Wave direction
$\boldsymbol{\theta}_k = (\theta_k, \varphi_k)$
Wavenumber **k**
Z
(θ_s, φ_s)
$\mathbf{r_s}$
Observation point
ϑ_s
a
φ_s
Y
X

Acoustic Scattering

- Scattering from general rigid object.

The problem of plane wave scattering from a rigid surface is to find that solution to the Helmholtz equation which satisfies

1. the Neumann boundary condition on the circular boundary $r = a$,
2. the boundary condition at infinity as when $r \to \infty$, the wave is a plane wave, and
3. the outward radiation condition.

By linearity, the solution has two parts: the incident plane wave and the scattered wave:

$$\phi = \phi_i + \phi_s$$

Acoustic Scattering

- Scattering of plane wave from direction **k** by rigid sphere.

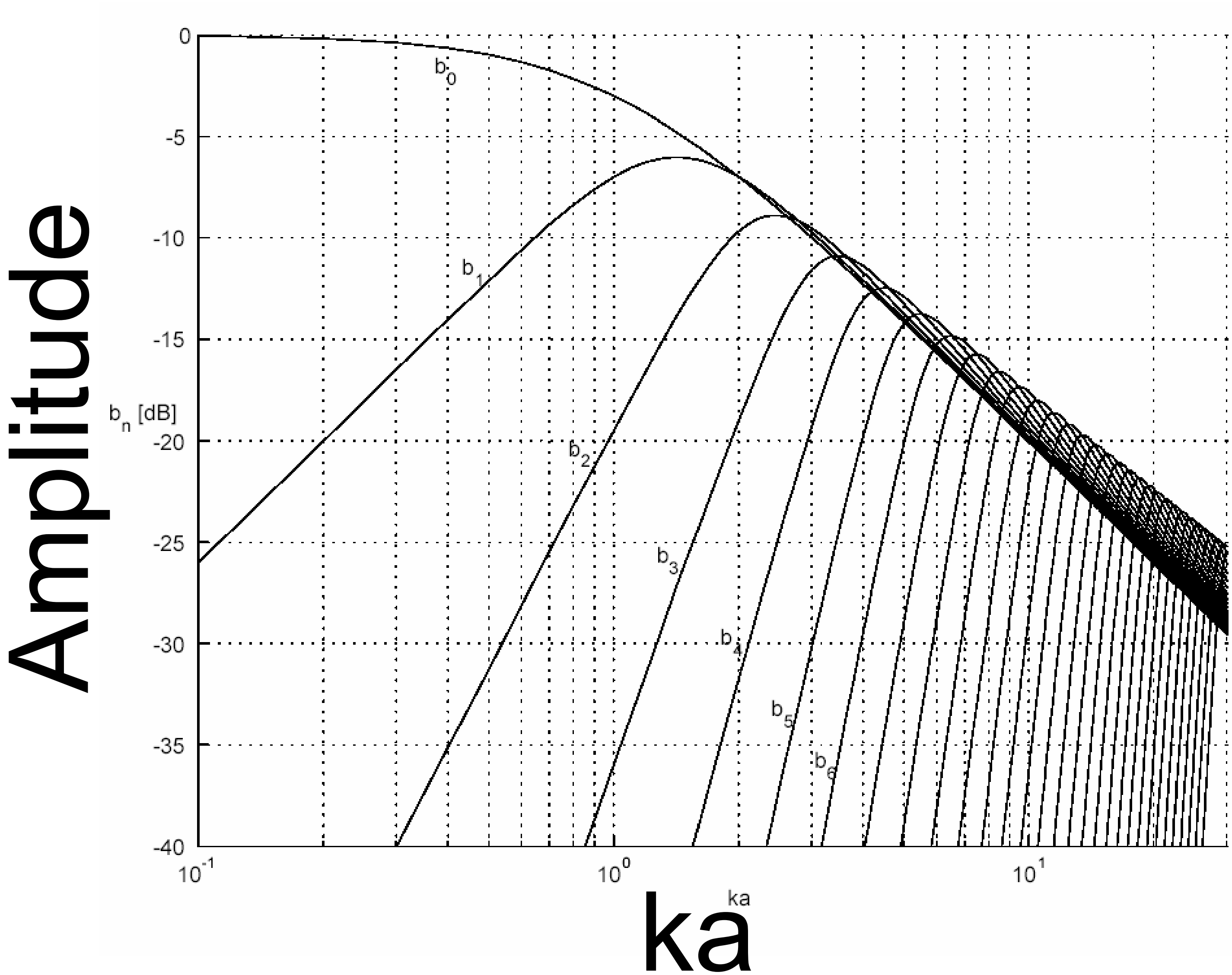
$$\psi\left(\boldsymbol{\theta}_s, \boldsymbol{\theta}_k, ka\right)$$

$$= \left[\psi_{in}\left(\mathbf{r}_s, \mathbf{k}\right) + \psi_{scat}\left(\mathbf{r}_s, \mathbf{k}\right)\right]\Big|_{r_s=a}$$

$$= 4\pi \sum_{n=0}^{\infty} i^n b_n(ka) \sum_{m=-n}^{n} Y_n^m(\boldsymbol{\theta}_k) Y_n^{m*}(\boldsymbol{\theta}_s),$$

$$b_n(ka) = j_n(ka) - \frac{j_n'(ka)}{h_n'(ka)} h_n(ka),$$

$b_n(ka)$: The Strength at n-th Order



3D Soundfield Sampling and Beamforming

- Sampling: the soundfield on the spherical surface is sampled continuously in ideal case, or discretely in practice.
- Beamforming (Spatial Filtering): the samples are weighted and combined smartly to keep the sound from desired directions, but suppress the sounds from all other directions.
- Beampattern (Spatial Response): the 3D response for unit magnitude plane wave from all directions. In ideal case, it is a delta function. In practice, it is a truncated version of its spherical harmonics expansion (the regular beampattern).

Principle of Spherical Beamformer (Orthogonal Decomposition)

Assigning the weight at each point:

$$W_{n'}^{m'}(\boldsymbol{\theta}_s, ka) = \frac{Y_{n'}^{m'}(\boldsymbol{\theta}_s)}{4\pi i^{n'} b_{n'}(ka)}$$

Using orthonormality of spherical harmonics:

$$\int_{\Omega_s} Y_n^{m*}(\boldsymbol{\theta}_s) Y_{n'}^{m'}(\boldsymbol{\theta}_s) d\Omega_s = \delta_{nn'}\delta_{mm'},$$

Output is:

The directional gain of the plane wave

$$\int_{\Omega_s} \psi(\boldsymbol{\theta}_s, \boldsymbol{\theta}_k, ka) W_{n'}^{m'}(\boldsymbol{\theta}_s, ka) d\Omega_s = Y_{n'}^{m'}(\boldsymbol{\theta}_k).$$

Principle of Spherical Beamformer

For example, the ideal beampattern looking at $\boldsymbol{\theta}_0$

$$F(\boldsymbol{\theta}, \boldsymbol{\theta}_0) = \delta(\boldsymbol{\theta} - \boldsymbol{\theta}_0)$$

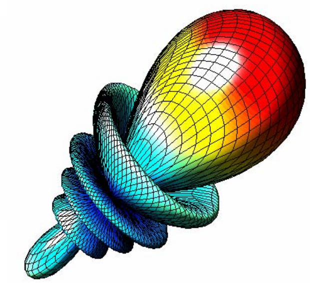
can be expanded into:

$$F(\boldsymbol{\theta}, \boldsymbol{\theta}_0) = 2\pi \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^{m*}(\boldsymbol{\theta}_0) Y_n^m(\boldsymbol{\theta})$$

So the weight for each point $\boldsymbol{\theta}_s$ is:

$$w(\boldsymbol{\theta}_0, \boldsymbol{\theta}_s, ka) = \sum_{n=0}^{\infty} \frac{1}{2 i^n b_n(ka)} \sum_{m=-n}^{n} Y_n^{m*}(\boldsymbol{\theta}_0) Y_n^m(\boldsymbol{\theta}_s).$$

In practice, with discrete spatial sampling, this is an finite number *N*.



N=5

Quadrature

- A quadrature formula provides layout and weights to obtain the integral.
- In spherical beamformer, this is a quadrature problem w.r.t. orthonormalities of spherical harmonics.

Quadrature weights

$$\frac{4\pi}{S}\sum_{s=1}^{S} Y_n^{m*}(\boldsymbol{\theta}_s) Y_{n'}^{m'}(\boldsymbol{\theta}_s) C_{n'}^{m'}(\boldsymbol{\theta}_s) = \delta_{nn'}\delta_{mm'},$$

$$(n = 0, ..., N_{eff}; m = -n, ..., n;$$

$$n' = 0, ..., N; m' = -n', ..., n'),$$

Bandlimit

Quadrature Examples

- Any quadrature formula of order N over the sphere should have more than $S = (N+1)^2$ quadrature nodes [Hardin&Sloane96][Taylor95].
- If the quadrature functions have the bandwidth up to order N, to achieve the exact quadrature using equiangular layout, we need $S = 4(N + 1)^2$ nodes [Healy94][Healy96].
- For special layouts, S can be much smaller. For example, for a spherical grid which is a Cartesian product of equispaced grid in φ and the grid in θ in which the nodes distributed as zeros of the Legendre polynomial of degree $N+1$ with respect to $\cos\theta$, we need $S = 2(N + 1)^2$ [Rafaely05].
- Spherical t-design: use special layout for equal quadrature weights. [Hardin&Sloane96]

Advantage and Disadvantage

- Advantage:
  - They achieve accurate quadrature results.
- Disadvantages:
  - They are only for strictly band-limited functions.
  - S is still large considering our quadrature function is the multiplication of two band-limited functions. This means that to achieve a beampattern of order *5* we need at least *144* microphones.

Practical Design of Discrete Microphone Array

- The most straightforward adaptation to the practical design is to position microphones "uniformly" so that we wish the following relations hold up to order *N*:

$$\frac{4\pi}{S}\sum_{s=1}^{S} Y_n^{m*}(\theta_s,\varphi_s)Y_{n'}^{m'}(\theta_s,\varphi_s)=\delta_{nn'}\delta_{mm'}$$

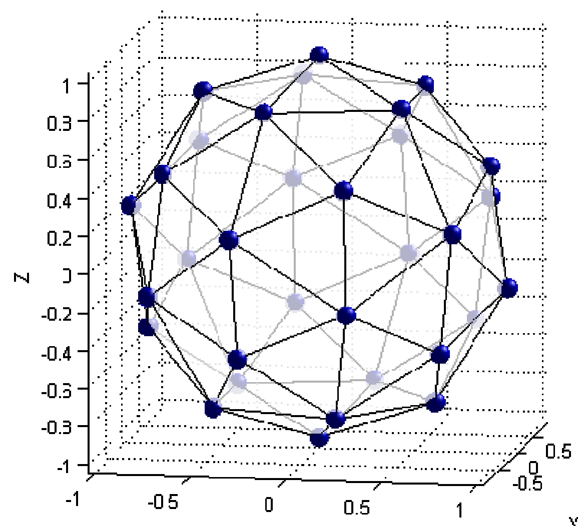
The number of microphones

The microphone angular positions

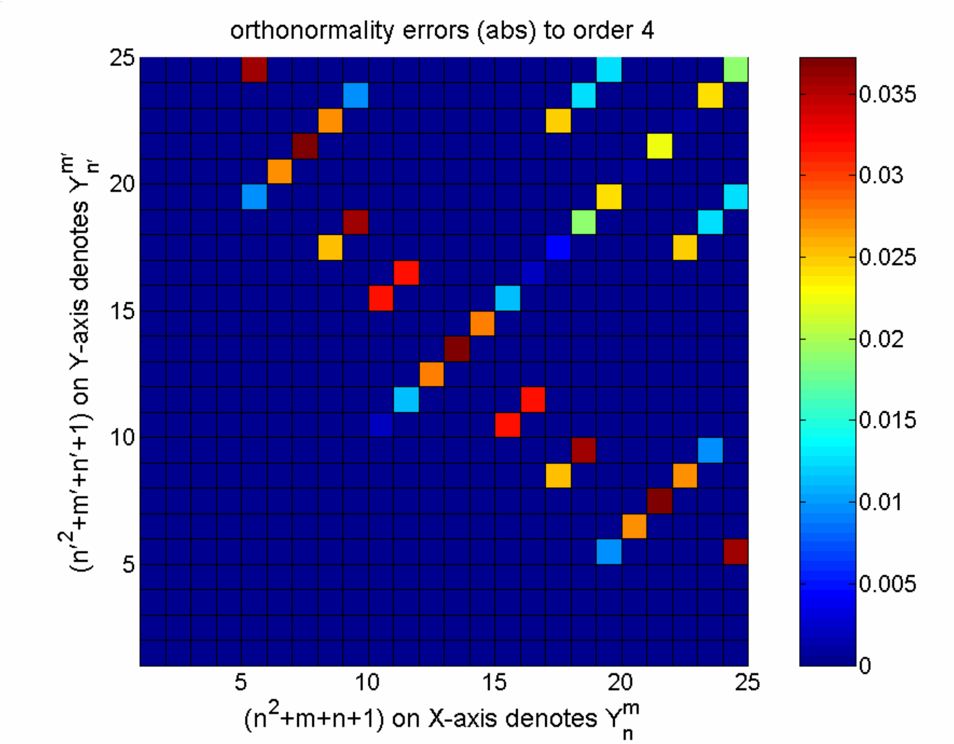
Regular and Semi-regular Layouts

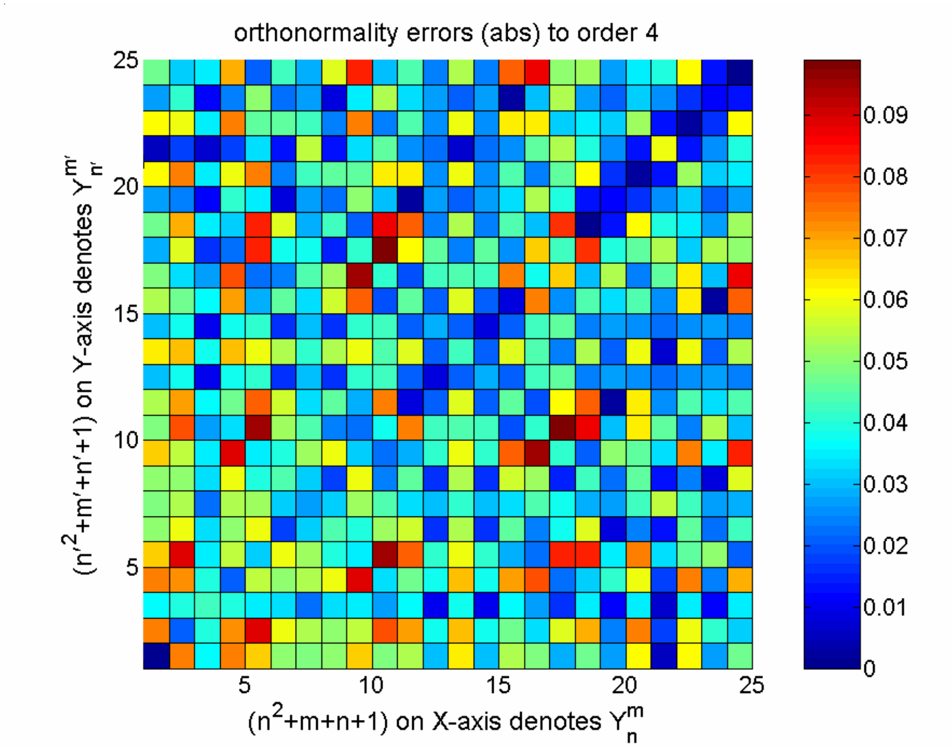
- Semi-regular polyhedrons can be used also such as the truncated icosahedron used by [Meyer&Elko02] to layout 32 microphones.
- Unfortunately, It can be proven that only five regular polyhedrons exist: cube, dodecahedron, icosahedron, octahedron, and tetrahedron [Steinhaus99, pp. 252-256]
- They are not available for arbitrary number of nodes.

The 32 nodes are the face centers of the truncated icosahedron



Orthonormality Errors





Using the 32-node layout and equal weights.

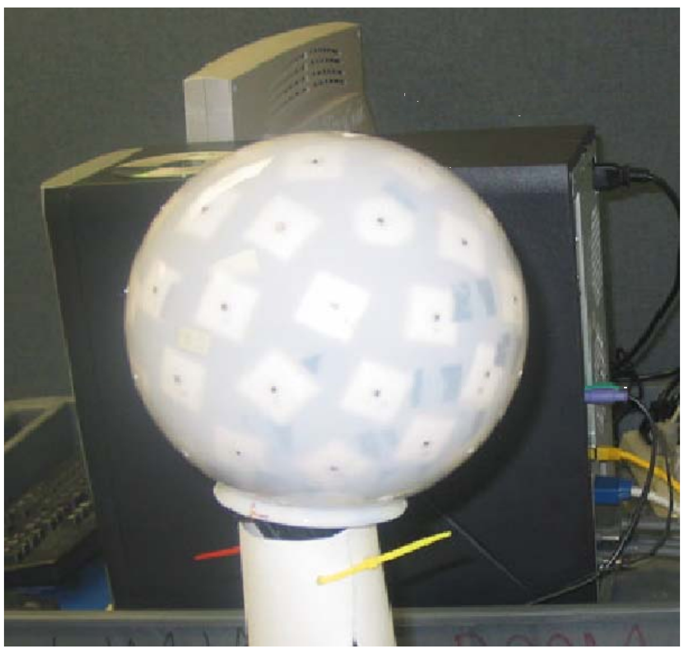
After removal of two nodes.

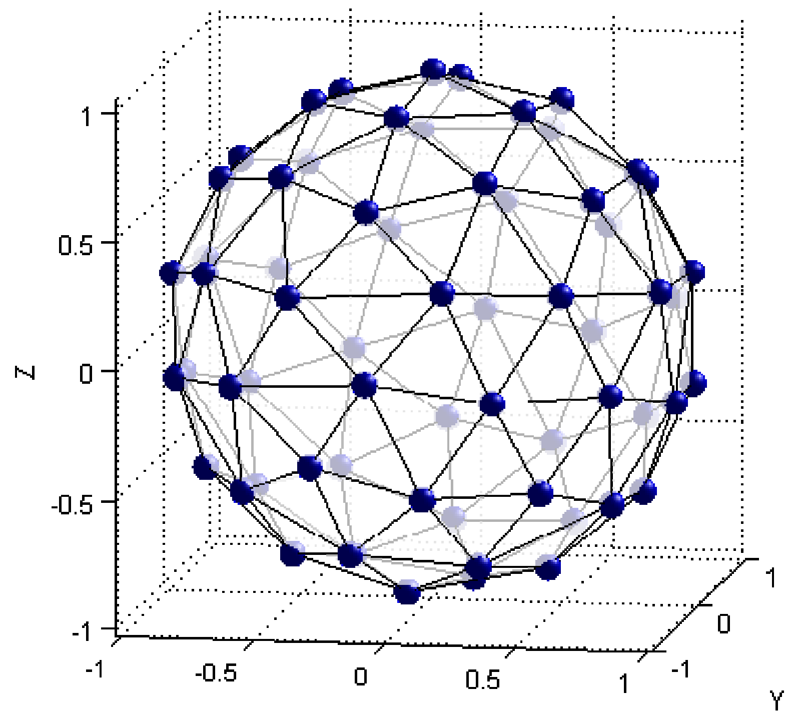
Achieve “Uniform” Layouts via Physical Simulation

- Achieve approximately “uniform” layouts for arbitrary number of nodes.
- By minimizing the potential energy of a distribution of movable electrons on a perfect conducting sphere.
- Then, one set of optimal quadrature coefficients are solved.
- [Fliege & Maier 1999].

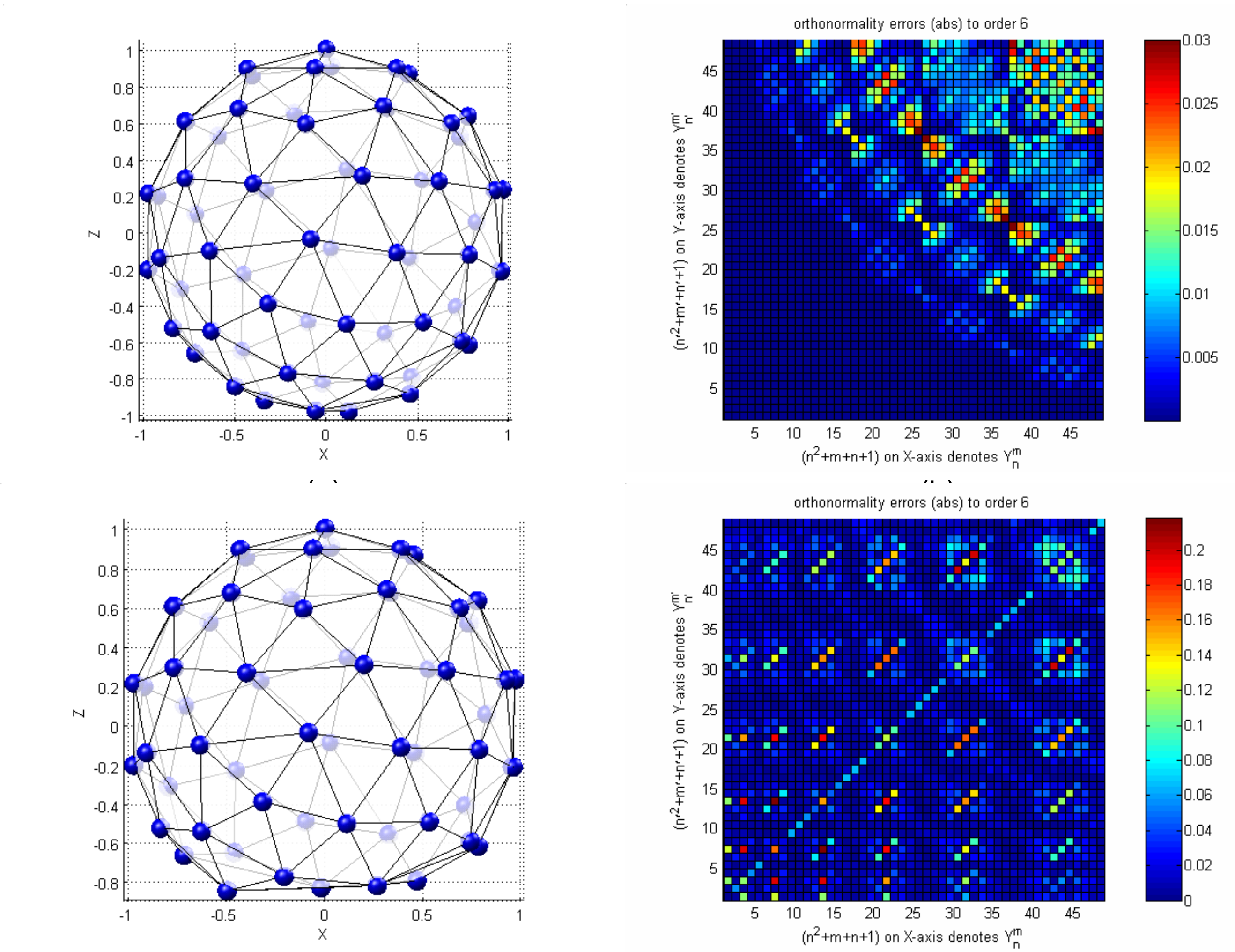
Our Spherical Microphone Array

- Our spherical array uses Fliege's 64 nodes with bottom four nodes missed.





Orthonormality Error (to order 6)



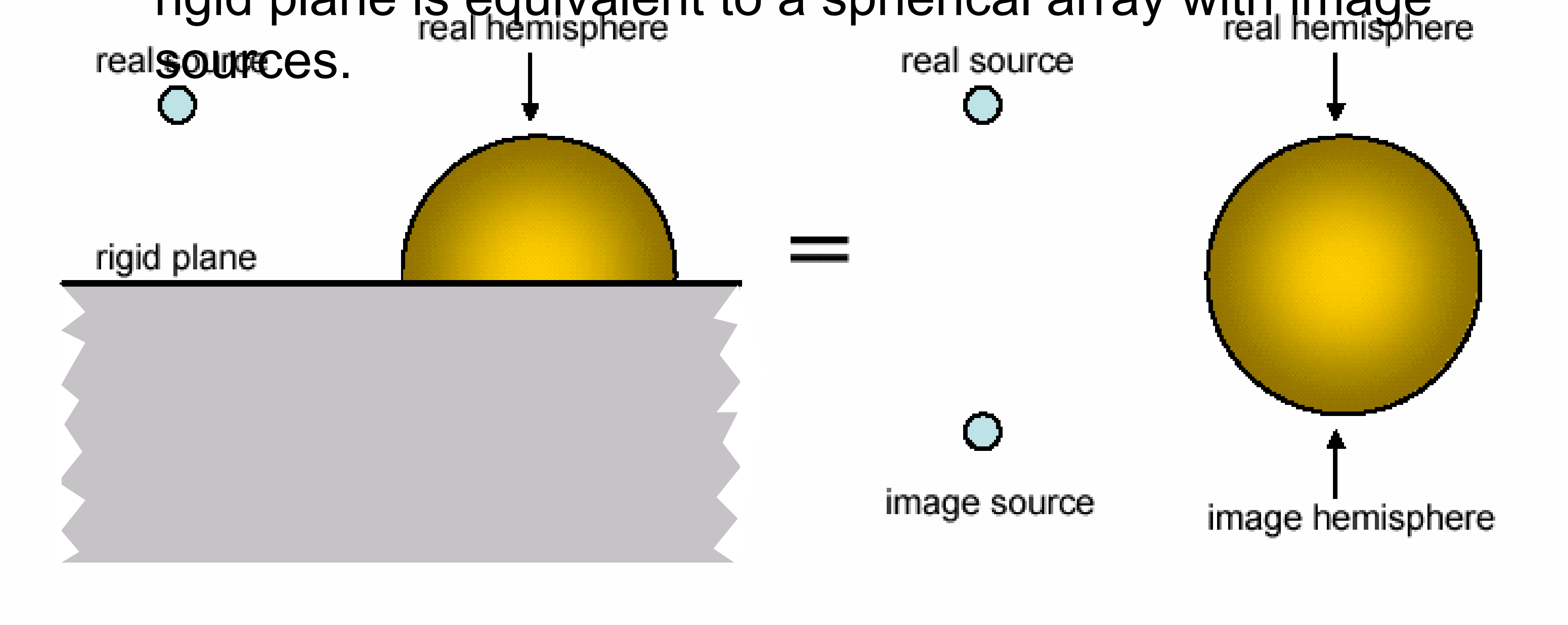
(a) (b)

Hemispherical Microphone Array

- In a conference table, the table surface is usually rigid and inevitably creates acoustic images.
- Given a specified number of microphones and a sphere of given radius, a hemispherical array will have a denser microphone arrangement, thereby allowing for analysis of a wider frequency range.
- Even in an acoustic environment without image sources, using a hemispherical array mounted on a rigid plane to create images may be appropriate since it provides higher order beampatterns.
- A hemispherical array is easier to build and maintain.

Acoustic Image Principle

- *Acoustic Image Principle*: The hemispherical array with a rigid plane is equivalent to a spherical array with image sources.



Symmetric and “Uniform” Layout

To obtain the hemispherical layout

1. We flip Fliege’s 64-node layout upside down and add it on the original 64 nodes to create an initial guess with 128 symmetric nodes.
2. We use the repelling electron simulation till all nodes are optimally separated.

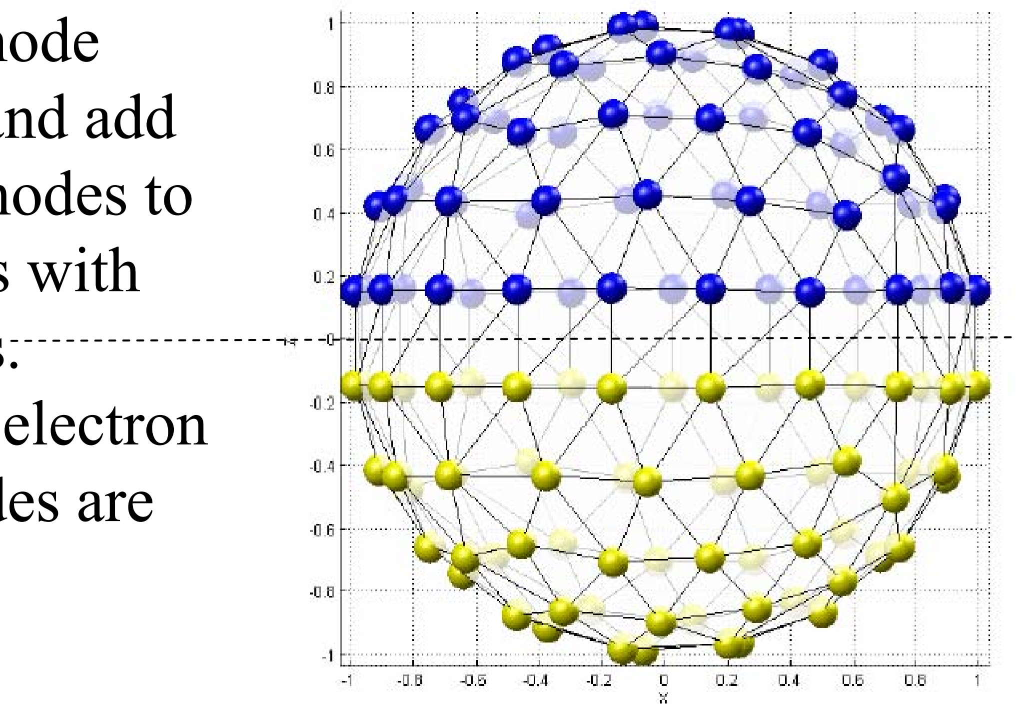
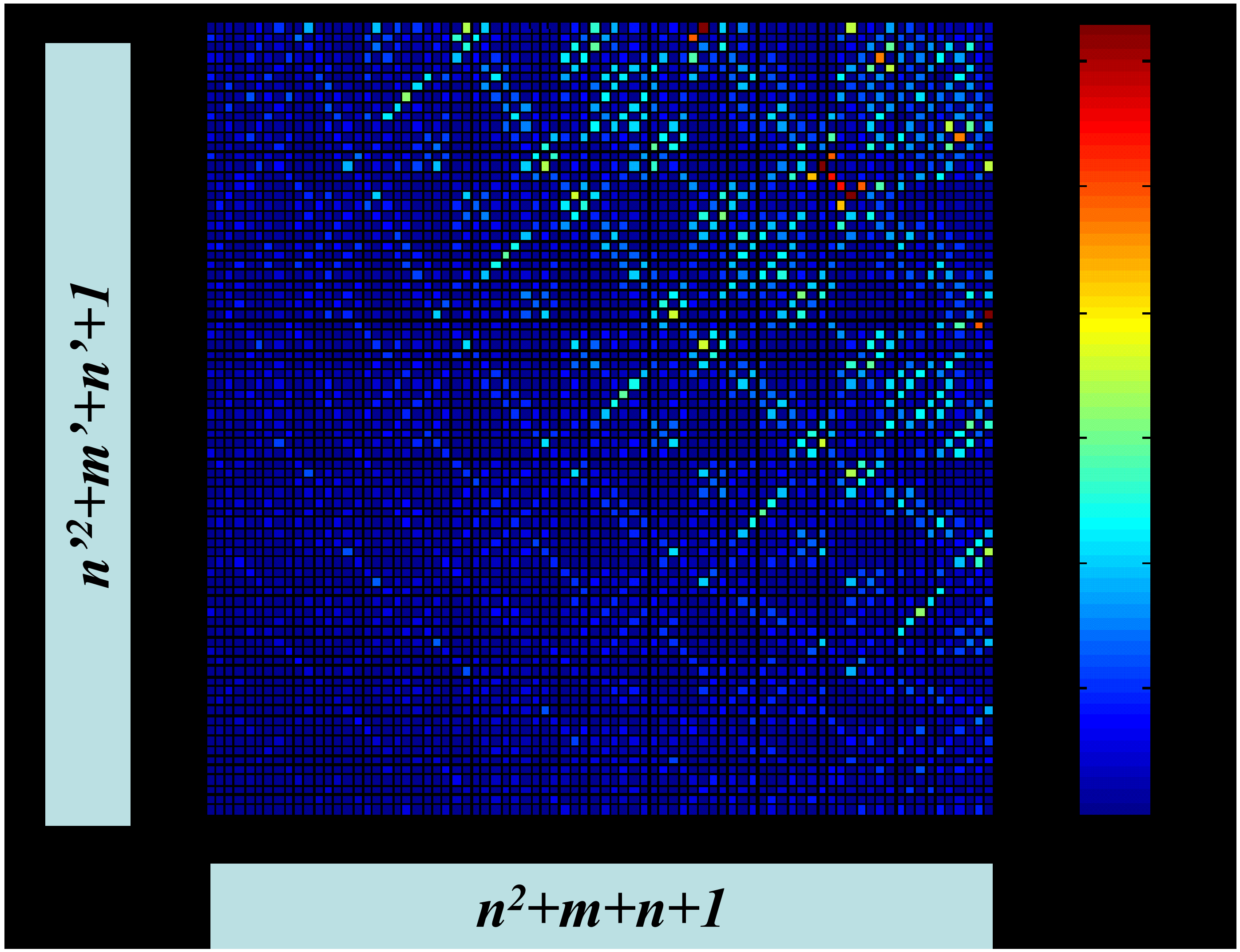


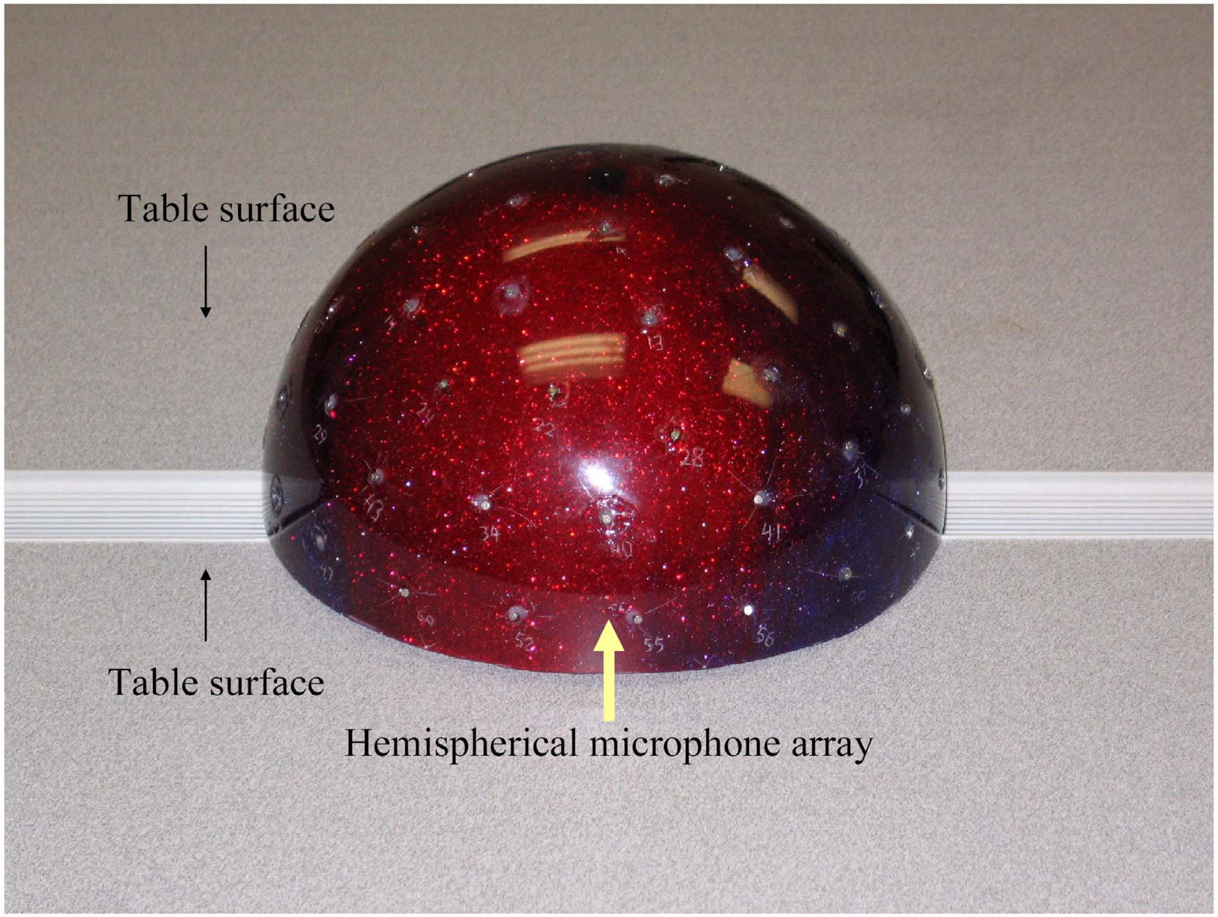
Figure 4: The symmetric and uniform layout of 128 nodes on a spherical surface. The blue (dark) nodes are for real microphones. The yellow (light) nodes are images.

Absolute Orthonormality Error Achieved

$$\int_S Y_{nm}\, Y_{n'm'}\, dS = \delta_{nn'}\, \delta_{mm'}$$



Prototype for Teleconference Applications



A hemispherical microphone array built on the surface of a half bowling ball. It uses 64 microphones. Its radius is 10.925cm.

Experiment Results (at 4KHz)

plane

plane

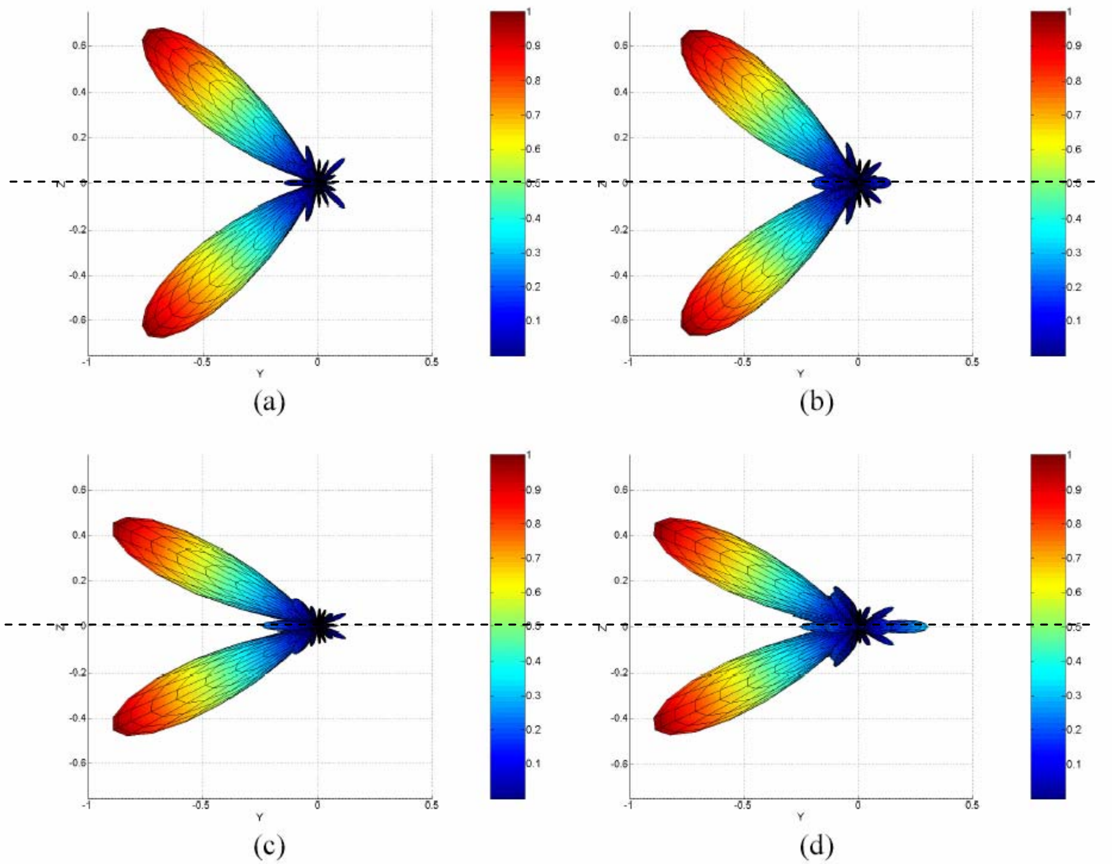
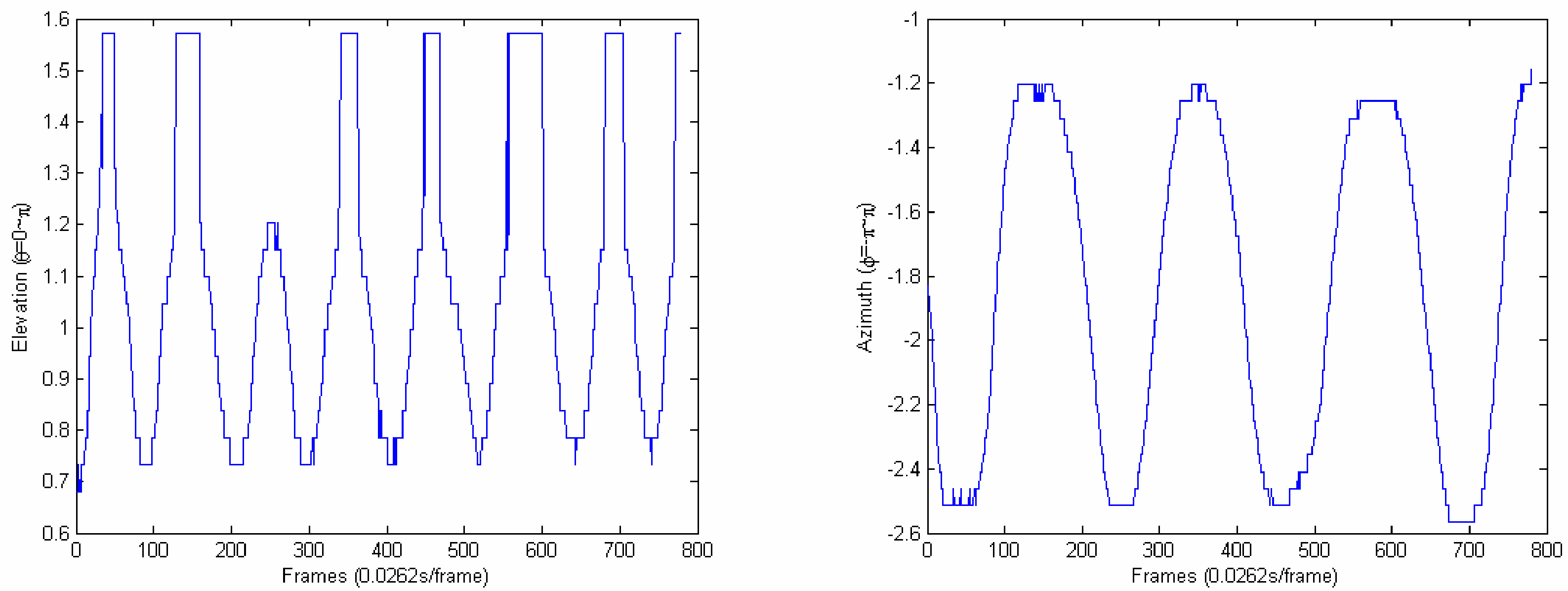


Figure 6: Simulation and Experimental results: (a) simulation of 3D scanning result with two sound sources, the beamformer is of order 8; (b) experimental result using the calibrated beamfomer of order 8; (c) simulation result after sound sources are moved; (d) experimental result using the same calibrated beamformer.

Experimental Results (Moving Sound Source Localization)



Moving sound source localization. Plots show the azimuth and elevation angles of the energy peak at each frame. The tracking is performed in the frequency band from 3kHz to 6kHz, with beamformers of order seven (3kHz~4kHz) and eight (4kHz~6kHz).