Politics Transformed? Electoral Competition under Ranked Choice Voting[*]

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Abstract

We compare multi-candidate elections under plurality rule versus ranked choice voting (RCV). In our framework candidates choose whether to pursue a narrow campaign that targets their base, or instead pursue a broad campaign that can appeal to the entire electorate. We present two main results comparing plurality and RCV. First, RCV can intensify candidates' incentives to target their core supporters at the cost of a broader appeal. Second, RCV may increase the probability that a candidate who would lose any pairwise contest nonetheless wins a multi-candidate contest.

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1. Introduction

Ranked Choice Voting (RCV) is the most publicly debated and rapidly expanding electoral reform in the United States. Rather than voting for a single candidate, voters under RCV can rank *multiple* candidates.[1] If any candidate wins a majority of first preferences, she is elected. If no candidate wins a majority of first preferences, the candidate with the fewest first preferences is eliminated, and each of her ballots transfers to the next-ranked candidate. The process repeats until a single candidate wins a majority of the remaining ballots.

RCV is widely employed in local and state elections, both in general elections and in the primaries of both major US political parties.[2] A notable recent example is New York City, which adopted RCV in its primary elections for both Mayor and City Council in 2019. The change was endorsed by a broad coalition of political actors and hailed as "a smart, tested reform that would make certain that New Yorkers elect candidates who have the support of a majority of voters" (New York Times 2019). More recently, Alaska adopted RCV for its state and federal elections through a voter initiative in 2020.

In this paper, we develop a new theoretical framework to analyze electoral competition under RCV, and we use it to examine widely held contentions about RCV's benefits relative to a plurality rule.[3]

The first contention we assess is that RCV encourages candidates to pursue a broad electoral appeal instead of focusing on their core supporters. The contention rests on the following logic: under plurality, a candidate only benefits from the support of voters that prefer her to every other candidate. This encourages a candidate to focus on mobilizing the narrow segment of voters that are most likely to prefer her over all other candidates—usually, an ideological, social, or ethnic base. Under RCV, by contrast, a candidate can benefit from

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[1] While variants of Ranked Choice Voting can also be used in multi-member districts (for a review, see Santucci 2021), we focus on the more common version with single-member districts—also called Instant Runoff. In this paper, we use RCV as a synonym of Instant Runoff, though the latter is technically a special case of the former.

[2] For a comprehensive list, see https://www.fairvote.org/.

[3] For a comprehensive summary of RCV's proposed benefits, see Cormack (2021).

the support of voters that do not like her the most. The prospect of winning voters' second preferences is expected to raise a candidate's relative benefit from broadening her platform in order to attract support from these voters, rather than focusing exclusively on her base. This contention is advanced by scholars (Horowitz 2004; Drutman 2020), pundits, and electoral reform advocates including *Common Cause* and *FairVote* in the United States and the United Kingdom's *Electoral Reform Society*.

The second contention we assess is that RCV better aligns electoral outcomes with voters' preferences—in particular, by weakening the prospect that a "Condorcet loser" wins a multi-candidate contest. A Condorcet loser is defined as a candidate that would lose in head-to-head competition with *any* other candidate. By definition, a Condorcet loser cannot win a two-candidate contest; she may nonetheless win a multi-candidate contest if voters divide their ballots across multiple candidates. RCV is expected to reduce that risk by allowing voters to transfer their ballots to other candidates in the event that their most preferred candidate is eliminated.

To examine these contentions, our model features three office-seeking candidates: a, b, and c, who compete for the support of an electoral divided into three groups: A, B, and C. Voters in groups A and B are collectively a majority, but voters in group C are a plurality. An example could be a district in which moderate (group A) and conservative (group B) Republicans are a majority of voters, but Democrats (group C) outnumber either of the two Republican groups.

Each candidate $j \in \{a, b, c\}$ chooses whether to pursue a broad campaign that appeals to all voters, or instead pursue a campaign that targets voters in corresponding group J, i.e., her "base", so that group A is a's base, group B is b's base, and minority group C is candidate c's base. In a distributive politics setting, the broad strategy represents an efficient spending allocation, versus a targeted policy that skews spending towards a candidate's core supporters or co-ethnics (e.g., Burgess et al. 2015). Alternatively, the policies could distinguish issues that all voters care about (e.g., the economy) from narrow issues that tend to excite only that candidate's base (e.g., culture wars). A candidate's base is defined as the voters that she can exclusively target, possibly reflecting differences in the candidates' clientelistic networks (Dixit and Londregan 1996), or their credibility on ethnic or ideological issue cleavages (Robinson

and Torvik 2005).

After the candidates choose policies, an aggregate preference shock is realized, and voters choose whether to turn out or abstain. Under plurality, a voter that turns out casts a ballot for a single candidate; under RCV, a voter that turns out chooses whether to rank one or more candidates. Voting decisions are guided by a simple heuristic based on *abstention due to alienation*, developed theoretically in Hinich, Ledyard and Ordeshook (1972), Callander and Wilson (2007), and Zakharov (2008). In this heuristic, a voter always ranks candidates according to her sincere preference, but she only awards a preference to candidates she likes enough. Implied turnout patterns find empirical support in plurality rule elections (Adams, Dow and Merrill 2006, Stewart 2020); we document evidence—including from our own voter survey conducted during the 2020 New York Democratic mayoral primary—that our model matches real-world RCV voting patterns.[4]

Candidates' policy platforms can therefore *persuade* voters to support them, but they can also *mobilize* turnout. Broad policies are more effective at changing voters' preferences over the candidates, but narrowly targeted policies are more effective at turning out a candidate's core supporters.

Under plurality, a candidate wins if and only if she receives more votes (first preferences) than *every* other candidate. In our three-candidate RCV setting, a candidate wins if and only if (a) no other candidate wins a strict majority of first preferences, (b) she herself does not win the fewest first preferences, and finally (c) her combined first and second preferences exceed those of whichever candidate does not receive the fewest first preferences. Two facts about RCV follow: winning second preferences from voters that do not like a candidate the most is valuable, and winning voters' first preferences may not be necessary to benefit from their support.

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[4] Our approach to voting behavior contrasts with formulations in which voters condition their choice on the relative prospects of pivotal events. The strategic and computational burden such behavior imposes on voters leads scholars to question its plausibility in real-world plurality rule elections (Van der Straeten, Laslier, Sauger and Blais 2010). This burden intensifies under RCV: in a three-candidate contest, the set of pivotal events expands from three under plurality rule to twelve under RCV (Eggers and Nowacki 2021).

Our paper's main insight is to show that these two facts need *not* encourage a candidate to pursue a broad strategy; they may have the opposite effect and intensify a candidate's incentive to target her base.

The key intuition underlying our result is that the possibility of winning second preferences under RCV changes how the candidates pursue *first* preferences. Under plurality, a candidate needs to win more first preferences than every other candidate. Under RCV, a candidate may instead focus on winning more first preferences than candidates *whose voters are likely to rank her second*. By defeating these candidates, she accrues a valuable dividend of second preferences that can strengthen her hand against the other candidates. Even if a candidate only wins first preferences from her core supporters, her ability to benefit from the second preferences of voters that do not rank her first may *reduce* her urgency from trying to capture these voters' first preferences with a broad campaign.

As a consequence, we unearth broad circumstances in which RCV intensifies the candidates' incentives to target their core supporters and eschew broad campaigning strategies. In particular, this arises in political contexts characterized either by high partisanship, or low baseline participation in elections. Ironically, these are the contexts in which reform advocates argue that RCV is most urgently needed.

We then turn to RCV's impact on the alignment of electoral outcomes with voter preferences. In our model, minority candidate c is a "Condorcet loser" whenever partisanship between the majority groups A and B versus the minority group C is sufficiently large. As Grofman and Feld (2004) observed, RCV necessarily weakens the Condorcet loser's victory prospects when the candidates' strategies (i.e., the set of alternatives) are fixed. We nonetheless show that the candidates' strategies under RCV can shift to an extent that the Condorcet loser enjoys a *strictly higher* probability of winning, relative to plurality rule.

To understand why, recognize that under either plurality or RCV, a Condorcet loser wins only when the majority of voters that oppose her divides between the remaining candidates. When the majority divides under plurality, candidates draw votes (i.e., first preferences) from one another. RCV alleviates the majority's vote-splitting problem by allowing voters to transfer their ballots via second preferences to other candidates. In fact, second preferences are *irrelevant* when the majority unites behind a single candidate. RCV therefore benefits a major-

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ity of voters only if they disagree over their preferred candidate. This is the channel through which RCV is expected to reduce the risk that a Condorcet loser wins.

For the same reason, however, RCV benefits a candidate's election prospects only if the majority divides. These divisions are more likely when the candidates adopt differentiated policies. Relative to plurality, RCV therefore tends to insure candidates' electoral prospects against the vote-splitting problem to a greater extent when they pursue policies that are *less* likely to unite the majority. This can benefit a candidate's individual election prospects, but it may incentivize electoral strategies that divide the majority to such a degree that the Condorcet loser's victory prospects increase, relative to plurality.

Contribution. Electoral systems can be assessed according to their effect on voters, and their effects on candidates. Existing work almost exclusively studies effects on voters, presuming that the set of alternatives (i.e., candidates and their policies) is fixed. In that vein, existing work highlights both experimentally (Van der Straeten et al. 2010) and computationally (Eggers and Nowacki 2021) that RCV can attenuate voters' incentives to cast strategic ballots, relative to plurality rule.

Two papers study policy outcomes and the number of candidates under RCV in a spatial model of elections: Callander (2005) and Dellis, Gauthier-Belzile and Oak (2017). Both characterize equilibria with two candidates who locate at polarized platforms: polarization is bounded by the threat of another candidate contesting the election at a centrist position between the two platforms.

In a citizen-candidate framework, Dellis, Gauthier-Belzile and Oak (2017) show that this bound is tightest under RCV. The reason is that a centrist entrant wins so long as she doesn't receive the *least* first preferences, since her centrist platform wins every voter's second preference. Under plurality, by contrast, a centrist entrant wins only if she receives the *most* first preferences. The authors conclude that RCV sustains less policy polarization than the plurality rule. Callander (2005) characterizes a continuum of equilibria in his office-motivated Downsian framework, highlighting the co-existence of equilibria with full median convergence under RCV, alongside equilibria with polarized platforms. Under plurality, by contrast, median convergence with three or more candidates cannot be supported (Cox 1987).

Because all voters turn out and fully utilize their ballots in both papers, and because candidates are differentiated solely by platforms, these frameworks do not address how candidates use their policy commitments to mobilize core supporters versus moderates. While our three-candidate framework abstracts from the question of how *many* candidates can be supported under RCV, both Callander (2005)'s and Dellis, Gauthier-Belzile and Oak (2017)'s analysis with endogenous candidacy highlights the stability of three-or-fewer candidate competition under RCV. This is also consistent with evidence from real-world elections documented in Jesse (2000) and Farrell and McAllister (2006).

2. Model

Electorate. A continuum of voters divide between a *majority* of mass one, and a *minority* of mass $\gamma < 1$. The majority-minority divide is most naturally interpreted as ideological or partisan, e.g., liberals versus conservatives, though it could also be an ethnic or religious cleavage. The majority further divides into two groups: group A of mass α, and group B of mass $1 - \alpha$. We refer to the mass γ minority as group C.

Assumption 1. $1 > \gamma \geq \alpha \geq .5$.

While $1 > \gamma$ states that the majority collectively outweighs the minority, $\gamma \geq \alpha$ states that the minority is a plurality. Without loss of generality, $\alpha \geq .5$ captures imbalances in the size of the majority groups A and B.

Policies. There are two kinds of policy: a *broad* policy g that gives all voters a common benefit, and a *targeted* policy t^I that only benefits voters in group $I \in \{A, B, C\}$. Thus, a voter in group I's benefit from policy p is

$$u^I(p) = \begin{cases} 1 & \text{if } p = t^I \\ u & \text{if } p = g \\ 0 & \text{otherwise.} \end{cases}$$

Assumption 2. $1 > u > \alpha$.

$1 > u$ states that a voter prefers her group's targeted policy over the broad policy. However, $u > \alpha$ implies that the broad policy generates higher (average) welfare amongst the majority

than a targeted policy, i.e., it implies that $\alpha u + (1 - \alpha)u > \alpha 1$.

Candidates. There are three purely office-seeking candidates: a, b, and c. Each candidate j simultaneously chooses a policy $p_j \in \{g, t^J\}$. That is: j can either offer the broad policy, or target voters in corresponding group J. Because candidate j can exclusively target group-J voters, we call that group her "*base*". That is: group A is a's base, group B is b's base, and minority group C is candidate c's base.

Payoffs. If either majority candidate $j \in \{a, b\}$ wins the election with policy p_j, a group-I voter's payoff is

$$U_j^I(p_j, \tau, \theta) = u^I(p_j) + \tau_j - \theta \mathbb{1}_{\{I=C\}}. \tag{1}$$

Here, τ_j is an aggregate preference shock in favor of majority candidate j. Letting τ denote a random variable distributed according to $T(\tau)$ on $\left[-\frac{1}{2}, -\frac{1}{2}\right]$, we set $\tau_a = \tau$ and $\tau_b = -\tau$. Thus, τ is a shock in favor of candidate a, and against candidate b. The shock captures developments that unfold over the course of the election: for example, changes in the candidates' reputation after a debate, or shifts in the popularity of their distinct positions on other policy issues.

The last term $-\theta$ in (1) reflects a loss that the minority suffers whenever a candidate from the majority wins. It could reflect partisan mis-alignment between the conservative majority and the liberal minority, in our earlier example.

Finally, if the minority candidate c implements policy c, a group-I voter's payoff is

$$U_c^I(p_c, \theta) = u^I(p_c) - \theta \mathbb{1}_{\{I=A,B\}}. \tag{2}$$

Throughout the analysis, we assume the following preference monotonicity.

Assumption 3. $\theta > \frac{u}{2}$.

We later verify that this condition is necessary and sufficient for the following property: if a majority voter prefers a over b, then she also prefers a over c. Likewise, if a majority voter prefers b over a, then also prefers b over c. Note that the condition imposes no restriction on a voter's second-ranked candidate. That is, it does not rule out that a majority voter prefers candidate c to one of the other majority candidates.

Turnout and Voting. Voters care about their participation in elections. We also assume they

rank candidates sincerely but may 'abstain due to alienation'. Specifically, each voter has an idiosyncratic *reservation utility*, η, uniformly distributed on $[\rho, \rho + \phi^{-1}]$. A voter casts a preference vote for candidate $j \in \{a, b, c\}$ in order of preference if and only if her payoff from the candidate of U_j^I exceeds η.

Timing. The timing unfolds as follows.

1. The candidates simultaneously select policies.

2. Nature draws the random variable τ.

3. Voters observe the policies and τ's realization, and make their decisions.

4. The winning candidate implements her policy, and payoffs are realized.

The solution concept is Nash Equilibrium, in which we further impose that no candidate plays a weakly dominated strategy. We assume that (1) $\phi < \frac{2}{3-2\rho}$, (2) $\rho < u/2$, and (3) $\rho > \rho_{\min} \equiv -\frac{2(\gamma-\alpha)+1-u}{2(1-\gamma)}$. Restrictions (1) and (2) ensure interior voter turnout from all groups, and (3) ensures that candidate c wins with positive probability under at least one strategy profile under both systems. We further impose indifference and tie-breaking rules that entail no loss of generality.[5]

Interpreting the Modeling Framework

We briefly discuss some of our modeling assumptions.

Divided Majority. Our restrictions on preferences resemble a classical 'Divided Majority' setting, but with the novelty that the candidates in our setting strategically choose policies, rather than confronting a fixed set of alternatives, as in existing models. While our preference restrictions serve primarily to facilitate analysis, we contend that our setting is important for at least three reasons.

First, it is central to positive and normative analyses of election systems—famously, Borda

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[5] For any group-I voter with reservation utility r: if $U_j^I = r$, the voter abstains from ranking the candidate; if $U_a^I = U_b^I > r$, and $I \in \{A, C\}$, the voter ranks a ahead of b; otherwise, she ranks b ahead of a; if $U_j^I = U_c^I$ for $j \in \{a, b\}$, and $I \in \{A, B\}$, the voter ranks j ahead of c; otherwise, she ranks c ahead of j. Any ties are resolved in alphabetical order: in favor of candidate a, followed by b, followed by c.

([1781](#)) and [Condorcet](#) ([1785](#)), but also in contemporary work by [Myerson and Weber](#) ([1993](#)), [Piketty](#) ([2000](#)), [Martinelli](#) ([2002](#)), [Dewan and Myatt](#) ([2007](#)), and [Myatt](#) ([2007](#)). [Bouton and Gratton](#) ([2015](#)) nest the divided majority in a general set of voter preferences. In addition to the novelty of endogenizing the set of alternatives, our preference restrictions are weaker than almost all existing models in that we do not assume that voters in the majority like candidate c the least. On the contrary, our assumptions allow minority candidate c to choose policies that secure some majority voters' second preferences.

Second, our setting matches many real-world elections. The papers cited offer an abundance of historical examples, but Alaska's August 2022 special congressional election also fits. Presuming—as we do—that voters' ballots reflect their sincere preference orders, a majority of voters favored a Republican: 31% of ballots ranked Palin (R) first, 29% ranked Begich (R) first, and 40% ranked Peltola (D) first; further analysis of second preferences concludes that Begich was the Condorcet winner. Nonetheless, vote-splitting and ballot exhaustion amongst Republicans likely facilitated Peltola's victory despite RCV's potential benefits.

Third, arguments about RCV's benefits over plurality seem especially compelling in a divided majority context. In the absence of preference cycles, second preferences matter—and *only* matter—when a group of voters that are collectively a majority divide over which candidate they like the most. Because broad policies help *any* candidate (including minority c) win second preferences even from voters that do not like her the most, and therefore should be strategically compelling under RCV. The possibility to cast second preferences should also alleviate the risk of electing a Condorcet loser—if one exists—under RCV, relative to plurality. Fixing the candidates' strategies, our model confirms this; nonetheless, electoral strategies are endogenous, and thus one must have a theory about how those strategies will change.

Voting Behavior. We assume voters cast their ballots sincerely. [Eggers and Nowacki](#) ([2021](#)) highlight the complexity of strategic voting incentives under RCV; indeed, there is presently no game-theoretic formulation. Sincere voting yields necessary tractability to maintain our key and novel focus on strategic candidates.

We also assume that voters can abstain both from turning out, and from fully utilizing their ballots. Turnout is a critical margin in real-world elections ([Hall 2015](#)). Under plurality rule, the 'reservation utility' could be interpreted as cost of voting, as in models that share

our perspective that voters abstain due to alienation, such as Hinich, Ledyard and Ordeshook (1972), Callander and Wilson (2007), and Zakharov (2008). Under RCV, of course, there is no marginal cost of ranking additional candidates. Nonetheless, many voters do *not* fully utilize their ballots. For example, in our own exit poll of New York voters during the 2021 Democratic primary, 52% of voters reported that they did not rank every candidate. Of those voters that ranked only one candidate, 67% reported the reason was "*that was the only candidate I liked*". This is precisely the behavior our voting heuristic captures.

3. Plurality Rule

Under plurality rule, voters that turn out can express a single preference for their most preferred candidate. Two preliminary lemmas simplify our analysis. The first lemma follows from our preference monotonicity restriction in Assumption 3 that $\theta > u/2$, and it applies under both plurality and RCV.

Lemma 1. *For any $p = (p_a, p_b, p_c)$: if a group-$I \in \{A, B\}$ voter prefers candidate a to candidate b, then she also prefers a to c; likewise, if she prefers b to a, then she also prefers b to c.*

This lemma implies that minority candidate c is never the most preferred candidate of any voter in group A or B. This means that regardless of her policy choice, candidate c wins first preferences only from her core supporters in the minority group C. Our second lemma follows from the first: c always weakly prefers to target her core supporters.

Lemma 2. *Under plurality rule, candidate c always weakly prefers $p_c = t^C$.*

Henceforth, we indeed presume that candidate c targets her core supporters with policy t^C. Later, we verify that this must be true in equilibrium. Under this strategy, minority c gives a value of 1 to a mass γ of voters in the minority. Since she is consequently always a minority voter's most preferred candidate, any such voter with a reservation utility $\eta < 1$ turns out to support her.[6] Recalling that η is uniformly distributed on $[\rho, \rho + \phi^{-1}]$, c's total first preferences

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[6] For any $p_a \in \{g, t^A\}$ and $\tau \in \left[-\frac{1}{2}, \frac{1}{2} \right]$, $U_c^C(t^C, \theta) > U_a^C(p_a, \tau, \theta)$ if $1 > u + \frac{1}{2} - \theta$, which is true since $u < 1$. The reader can verify similarly that for any $p_b \in \{g, t^B\}$ and $\tau \in \left[-\frac{1}{2}, \frac{1}{2} \right]$, $U_c^C(t^C, \theta) > U_b^C(p_b, \tau, \theta)$. Thus, voters in minority group C prefer c over each of a and b for any aggregate shock realization.

under plurality are therefore $\gamma\phi(1 - \rho)$.

We turn to the behavior of majority voters and majority candidates. A group-$I \in \{A, B\}$ voter prefers a over b if and only if

$$u^I(p_a) + \tau \geq u^I(p_b) - \tau \iff \tau \geq \frac{u^I(p_b) - u^I(p_a)}{2} \equiv \tau_{ab}^I(p_a, p_b). \tag{3}$$

Further, a group-$I \in \{A, B\}$ voter who prefers a turns out if and only if her value from a exceeds her idiosyncratic reservation utility, η. Lemma 1 yields candidate a's total first preferences, and thus her total votes under plurality rule:

$$v_a^f(p_a, p_b, t^C, \tau) \equiv \alpha\phi(u^A(p_a) + \tau - \rho)\mathbb{I}\{\tau \geq \tau_{ab}^A(p_a, p_b)\} + (1-\alpha)\phi(u^B(p_a) + \tau - \rho)\mathbb{I}\{\tau > \tau_{ab}^B(p_a, p_b)\}. \tag{4}$$

We similarly obtain b's total first preferences:

$$v_b^f(p_a, p_b, t^C, \tau) \equiv \alpha\phi(u^A(p_b) - \tau - \rho)\mathbb{I}\{\tau < \tau_{ab}^A(p_a, p_b)\} + (1-\alpha)\phi(u^B(p_b) - \tau - \rho)\mathbb{I}\{\tau \leq \tau_{ab}^B(p_a, p_b)\}. \tag{5}$$

Given platforms p_a and p_b, and c's platform t^C, we can define a critical realization of the aggregate shock that determines whether candidate $j \in \{a, b\}$'s first preferences exceed c's first preferences.

$$\tau_{\text{plu}}^{ac}(p_a, p_b, t^C) \equiv \inf\{\tau : v_a^f(p_a, p_b, t^C, \tau) \geq \gamma\phi(1 - \rho)\} \tag{6}$$

$$\tau_{\text{plu}}^{bc}(p_a, p_b, t^C) \equiv \sup\{\tau : v_b^f(p_a, p_b, t^C, \tau) \leq \gamma\phi(1 - \rho)\}. \tag{7}$$

Candidate a wins more first preferences than c whenever $\tau \geq \tau_{\text{plu}}^{ac}(p_a, p_b, t^C)$, and b wins more first preferences than c whenever $\tau \leq \tau_{\text{plu}}^{bc}(p_a, p_b, t^C)$. We can also define the critical shock realization above which a's first preferences exceed b's:

$$\tau^{ab}(p_a, p_b) \equiv \inf\{\tau : v_a^f(p_a, p_b, t^C, \tau) \geq v_b^f(p_a, p_b, t^C, \tau)\}. \tag{8}$$

Under the supposition that c targets her base, a pair of mutual (pure strategy) best responses

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for a and b under plurality is therefore $p^* = (p_a^*, p_b^*)$ such that

$$p_a^* = \arg\min_{p_a} \left\{ \max\{\tau_{\text{plu}}^{ac}(p_a, p_b^*, t^C), \tau^{ab}(p_a, p_b^*)\} \right\} \tag{9}$$

$$p_b^* = \arg\max_{p_b} \left\{ \min\{\tau_{\text{plu}}^{bc}(p_a^*, p_b, t^C), \tau^{ab}(p_a^*, p_b)\} \right\}. \tag{10}$$

In words: candidate a wins if and only if preference shock τ exceeds the largest of two thresholds: one corresponds to winning enough first preferences to defeat b and the other corresponds to winning enough first preferences to defeat c. So, a's objective is to weaken the most demanding of these two thresholds. The interpretation of b's objective is similar. A pair (p_a^*, p_b^*) satisfying (9) and (10), together with $p_c = t^C$, is a Nash equilibrium.

It turns out that candidate $j \in \{a, b\}$'s binding constraint is *always* to defeat c, and thus (9) and (10) can be simplified.

Lemma 3. *When $p_c = t^C$, for any (p_a, p_b): if and only if majority candidate $j \in \{a, b\}$'s first preferences exceed c's, then candidate j wins under plurality rule.*

The reason is that candidate c's base of mass γ exceeds either a's or b's base (mass $\alpha \leq \gamma$ and $1 - \alpha \leq \alpha$, respectively), and c offers her core supporters their highest possible policy payoff of 1. We can therefore simplify conditions (9) and (10):

$$p_a^* = \arg\min_{p_a} \tau_{\text{plu}}^{ac}(p_a, p_b^*, t^C) \;\; ; \;\; p_b^* = \arg\max_{p_b} \tau_{\text{plu}}^{bc}(p_a^*, p_b, t^C).$$

We start by analyzing conditions for a 'broad' equilibrium in which both majority candidates offer the broad policy g. Under that strategy, majority voters are indifferent between candidates a and b on policy grounds, since both candidates offer every voter the same policy payoff of u. Majority voters' first preferences therefore resolve on the aggregate shock, τ; if the shock is positive they unanimously favor a and if the shock is negative they unanimously favor b:

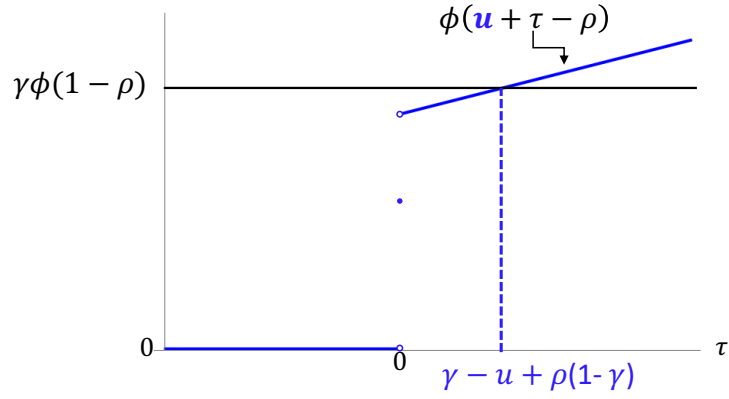$$\tau_{ab}^A(g, g) = \tau_{ab}^B(g, g) = 0.$$

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Figure 1 – Candidate a's total first preferences $v_a^f(g, g, t^C, \tau)$, and their intersection with c's first preferences of $\gamma\phi(1 - \rho)$.

So, a's first preferences at this policy profile are:

$$v_a^f(g, g, t^C, \tau) = \begin{cases} 0 & \text{if } \tau < 0 \\ \alpha\phi(u + \tau - \rho) & \text{if } \tau = 0 \\ \alpha\phi(u + \tau - \rho) + (1 - \alpha)\phi(u + \tau - \rho) \, \big(= \phi(u + \tau - \rho) \big) & \text{if } \tau > 0. \end{cases} \quad (11)$$

Figure 1 plots expression (11): it shows that a wins more votes than c—and therefore the election—if and only if a's combined support from the majority exceeds c's support from the minority:

$$\phi(u + \tau - \rho) > \gamma\phi(1 - \rho) \iff \tau > \max\{0, \gamma - u + \rho(1 - \gamma)\} \equiv \tau_{\text{plu}}^{ac}(g, g, t^C). \quad (12)$$

The requisite threshold increases as minority group-C's mass γ increases, and decreases as majority voters' common benefit from the broad policy u increases. This reflects that higher u raises majority voters' value from the broad policy, and their turnout increases with the value they get from their preferred candidate.

Recall that reservation utility η is uniformly distributed on $[\rho, \rho + \phi^{-1}]$. Larger values of ρ reflect greater voter apathy or disengagement. Greater apathy lowers voter turnout in both the majority *and* the minority—in Figure 1, larger ρ lowers both the blue *and* black lines. Larger ρ therefore lowers *every* candidate's total first preferences. However, the decrease in the candidates' turnout is proportional to the mass of voters that like her the most. Higher

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apathy reduces a's turnout from the united majority proportional to its mass of 1, but reduces c's turnout from the minority proportional to its mass γ: hence, higher apathy ρ raises threshold $\tau_{\text{plu}}^{ac}(g, g, t^C)$ defined in (12) by $1 - \gamma$. With reference to Figure 1, this means that the blue line falls faster than the black line as ρ increases. Thus, more apathy in the electorate reduces advantages stemming from winning support from a larger block of voters.

Suppose, alternatively, that candidate a targets her core supporters with policy t^A. This means that she gives a policy payoff of 1 to group-A voters, and zero to group-B voters, while candidate b's broad strategy gives u to all voters. As a consequence, the majority divides for intermediate preference shocks:

$$\tau_{ab}^A(t^A, g) = \frac{u - 1}{2}, \quad \tau_{ab}^B(t^A, g) = \frac{u}{2}.$$

Candidate a's total first preferences—highlighted by the red line in Figure 2—are therefore

$$v_a^f(t^A, g, t^C \tau) = \begin{cases} 0 & \text{if } \tau < \frac{u-1}{2} \\ \alpha\phi(1 + \tau - \rho) & \text{if } \frac{u-1}{2} \leq \tau \leq \frac{u}{2} \\ \phi\alpha(1 + \tau - \rho) + \phi(1 - \alpha)(0 + \tau - \rho) \ \big(= \phi(\alpha + \tau - \rho) \big) & \text{if } \frac{u}{2} < \tau. \end{cases}$$
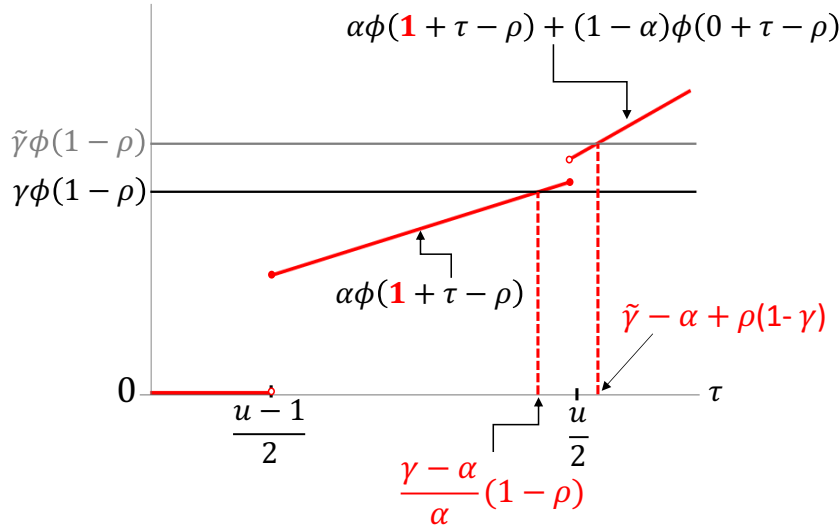
$$(13)$$

Figure 2 – Candidate a's total first preferences $v_a^f(t^A, g, t^C, \tau)$, and their intersection with c's first preferences for low (γ) versus high ($\tilde{\gamma}$) sizes of c's base.

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Depending on primitives, candidate a either defeats c for $\tau \in [(u-1)/2, u/2]$ where the majority divides, or instead defeats c for $\tau > u/2$ for which the majority unites behind a. Using the Figure, we infer that a wins more votes than c—and therefore the election—if and only if

$$v_a^f(t^A, g, t^C, \tau) \geq \gamma\phi(1-\rho)$$

$$\iff \tau \geq \min\left\{\frac{\gamma-\alpha}{\alpha}(1-\rho), \max\left\{\frac{u}{2}, \gamma-\alpha+\rho(1-\gamma)\right\}\right\} \equiv \tau_{\text{plu}}^{ac}(t^A, g, t^C). \tag{14}$$

So, given that b pursues a broad policy, candidate a maximizes her probability of election by campaigning broadly if and only if threshold $\tau_{\text{plu}}^{ac}(t^A, g, t^C)$ defined in (14) is weakly larger than threshold $\tau_{\text{plu}}^{ac}(g, g, t^C)$ defined in (12).

To understand when this condition holds, consider Figure 3, which presumes that c targets her base, and b pursues a broad campaign. The blue line plots a's first preferences when she campaigns broadly, i.e., it plots in blue expression (11); the red line plots a's first preferences when she instead targets her base, i.e., it plots in red expression (13). For very favorable shock realizations $\tau > u/2$, the figure highlights that a unites the majority regardless of her strategy, but nonetheless secures higher overall turnout by pursuing a broad policy. The reason is that,
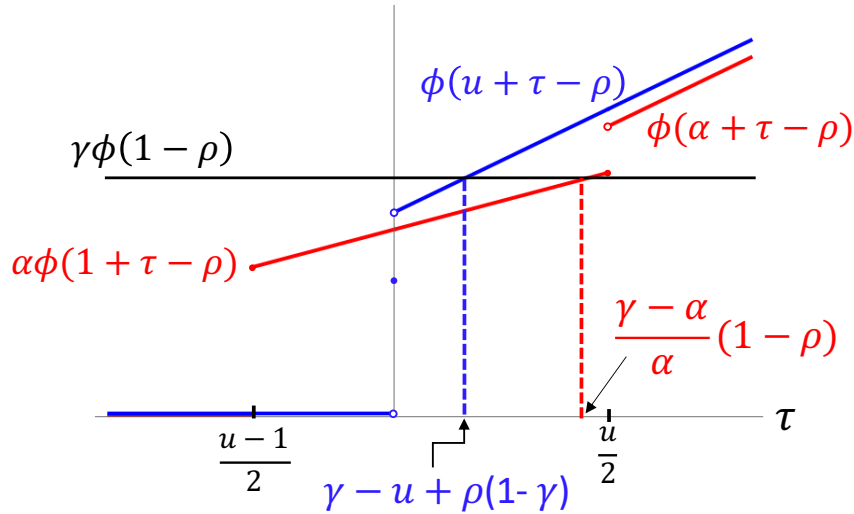

Figure 3 – Comparing a's total first preferences $v_a^f(p_a, g, t^C, \tau)$ for $p_a = t^A$ (red) and $p_a = g$ (blue).

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for these shock realizations, candidate a's total turnout is:

$$\alpha\phi(u^A(p_a) + \tau - \rho) + (1 - \alpha)\phi(u^B(p_a) + \tau - \rho) \propto \underbrace{\alpha u^A(p_a) + (1 - \alpha)u^B(p_a)}_{\text{average majority welfare from } p_a} + \tau - \rho,$$

and Assumption 2 that $u > \alpha$ implies that the broad policy maximizes majority welfare. In fact, for the parameters used in that example, a wins more votes under plurality for any positive shock realization $\tau > 0$. This raises the question: are there *any* circumstances in which a would prefer to target her core supporters?

The answer is *yes*. To see why, recall that higher apathy—reflected in higher ρ—lowers turnout across all groups, and therefore decreases every candidate's first preferences. In Figure 3, this amounts to a decrease in the black, red, and blue lines. However, each candidate's decrease in first preferences is proportional to the size of the voting bloc that supports her. This has two consequences.

First, majority candidate a's electoral advantage over minority candidate c when she unites the majority—increasing in the difference $1 - \gamma$—*decreases* with more apathy. The figure highlights the threshold above which a defeats c with the support of the majority: $\tau_{\text{plu}}^{ac}(g, g, t^C) = \gamma - u + \rho(1 - \gamma)$ *increases* in ρ. So, higher ρ effectively makes the blue line in Figure 3 fall faster than the black line.

Second, majority candidate a's disadvantage relative to candidate c in the event that a only wins votes from her core supporters—increasing in the difference $\gamma - \alpha$—*also decreases* with more apathy. The figure highlights the threshold above which a defeats c with the support of just her base: $\tau_{\text{plu}}^{ac}(t^A, g, t^C) = \frac{\gamma - \alpha}{\gamma}(1 - \rho)$ *decreases* in ρ. Intuitively, higher ρ makes the intermediate segment of Figure 3's red line fall slower than the black line.

The left-hand side of Figure 4 reproduces Figure 3: in low-apathy contexts, a is better-off marshaling the support of a united majority than trying to defeat minority c's larger base by turning out her own. The right-hand side nonetheless illustrates that in high-apathy contexts a's trade-off shifts in favor a targeted strategy. Indeed, comparison of (12) and (14) yields

$$\tau_{\text{plu}}^{ac}(t^A, g, t^C) \geq \tau_{\text{plu}}^{ac}(g, g, t^C) \iff \rho \leq 1 - \frac{\alpha}{\gamma}\frac{1 - u}{1 - \alpha} \equiv \overline{\rho}.$$
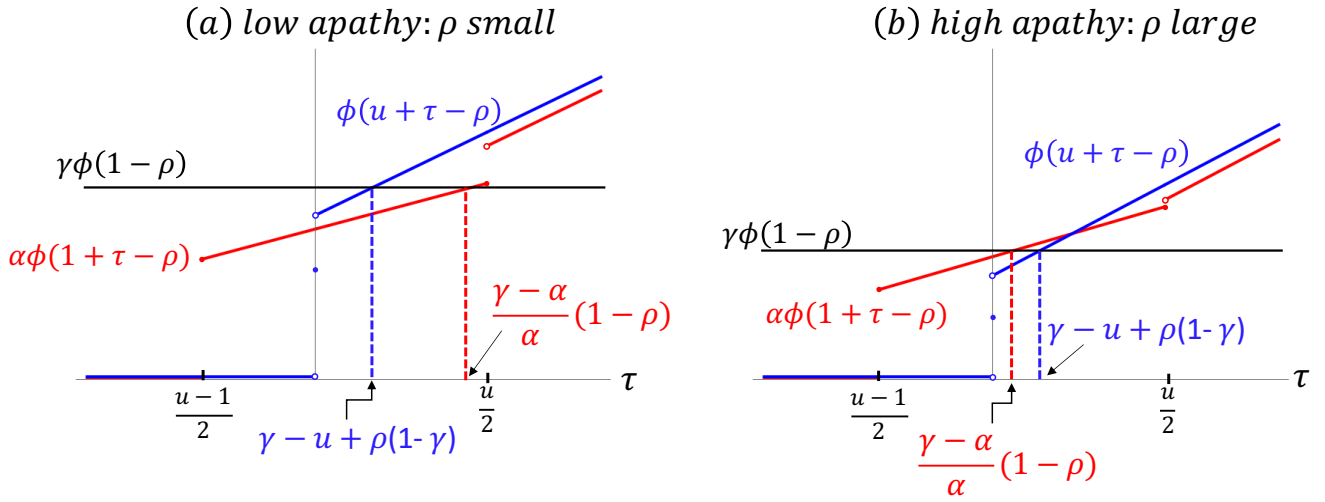
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Figure 4 – Incentives to target the base under plurality rule, for *low* versus *high* apathy. *Parameters*: $u = \gamma = .6$.

It is easy to verify that whenever advantaged a prefers a broad strategy, so does b. We therefore have our first result.

Proposition 1. *Under plurality, there exists an equilibrium in which both majority candidates offer the broad policy if and only if $\rho \leq \bar{\rho}$.*

In high-apathy contexts $\rho > \bar{\rho}$, the majority candidates do not converge on the broad strategy. In fact, advantaged majority candidate a *always* targets her core supporters with policy t^A. Candidate b's strategy depends on her core supporters' size of $1 - \alpha$ versus a's core supporters of mass $\alpha > 1/2$. If α is large, meaning that group B of mass $1 - \alpha$ is very small, candidate b maintains the broad strategy: she cannot hope to defeat minority c simply by relying on the support of her own base, group B. If, instead, α is close enough to one half, meaning that the majority is evenly divided between groups A and B, then candidate b also reverts to targeting her base.

Proposition 2. *There exists α^{plu} such that if and only if $\rho > \bar{\rho}$ and $\alpha > \alpha^{plu}$, the unique equilibrium under plurality is (t^A, g, t^C). Whenever $\alpha \leq \alpha^{plu}$, (t^A, t^B, t^C) is an equilibrium.*

Figure 5 identifies the equilibria characterized in Propositions 1 and 2 for different pairs (ρ, α). The following remark clarifies the scope for multiplicity.
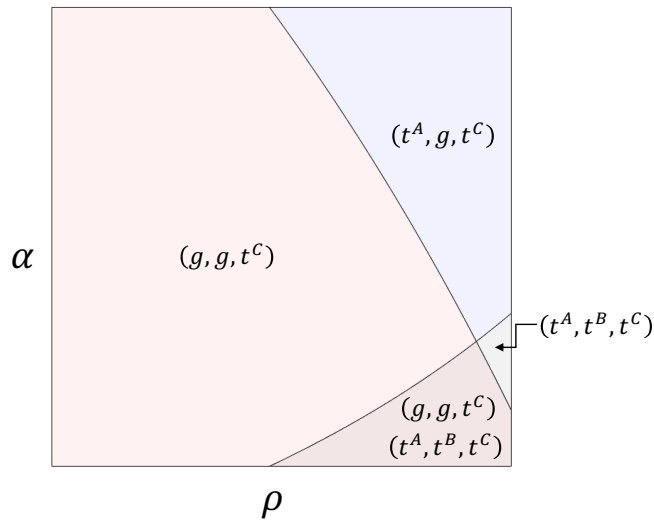
17

Figure 5 – Equilibria under plurality rule. *Parameters*: $u = \gamma = .6$.

Remark 1. *Under plurality, every pure strategy equilibrium in weakly undominated strategies is outcome-equivalent to an equilibrium characterized in Propositions 1 or 2, and a unique pure strategy equilibrium is the unique Nash equilibrium.*[7]

Figure 5 highlights that for some parameters, (g, g, t^C) *and* (t^A, t^B, t^C) can be sustained as equilibria. The logic is as follows: if candidate a targets voters in group A, she makes it harder for candidate b to contest group-A votes; even if b pursues a broad campaign voters in group A prefer b only for shock realizations that are very unfavorable to a, and therefore relatively unlikely. This encourages b to instead focus on mobilizing her own base.

When these equilibria co-exist, they can be ranked in a strong sense: average welfare is strictly higher in the (g, g, t^C) equilibrium than the (t^A, t^B, t^C) equilibrium, and both majority candidates enjoy a strictly higher probability of winning in the (g, g, t^C) equilibrium than the (t^A, t^B, t^C) equilibrium.

---

[7] The proof of this remark verifies that at most one other pure strategy equilibrium can exist, in which $p_a = p_b = p_c = g$. This equilibrium can be sustained only for parameters such that candidate c loses with probability one regardless of her campaigning strategy when $p_a = p_b = g$, i.e., for $\rho \leq \frac{u-\gamma}{1-\gamma}$. Since $\frac{u-\gamma}{1-\gamma} < \bar{\rho}$, this equilibrium is outcome-equivalent to the equilibrium characterized in Proposition 1.

Proposition 3. *Under plurality, when $\alpha < \alpha^{plu}$ and $\rho < \overline{\rho}$ each majority candidate $j \in \{a,b\}$'s probability of winning is* strictly lower *in the (t^A, t^B, t^C) equilibrium than in the (g, g, t^C) equilibrium.*

For these primitives, the (t^A, t^B) equilibrium therefore represents a classic coordination trap that is bad for both voters and the majority candidates. Nonetheless a divergent equilibrium is not always a coordination trap: as the Figure highlights, when $\rho > \overline{\rho}$ and $\alpha < \alpha^{plu}$ a divergent equilibrium is the *unique* equilibrium.

To summarize: if the electorate is not too apathetic, a convergent equilibrium exists in which both candidates pursue broad electoral strategies. Otherwise, the majority candidate with the larger base targets it. In that case, candidate b maintains a broad strategy if the imbalance within the majority is large; otherwise, she also reverts to a base strategy. Finally, even when voters are not too apathetic, the candidates may face a coordination trap: when imbalances in group size amongst the majority aren't too large, convergent and divergent equilibria co-exist.

4. Ranked Choice Voting

Under ranked choice voting (RCV), voters that turn out can express as many preferences as they want. A group-I voter turns out and assigns preferences to any candidate j whose value to that voter U_j^I exceeds that voter's idiosyncratic reservation utility of η.

The candidates' first preferences as a function of their campaigning strategies are the same under RCV as they were under plurality. Under RCV, however, a candidate might additionally win some *second* preferences from voters that do not rank her first.[8] Regardless of which candidate they like the most, group-$I \in \{A, B\}$ voters prefer a over c so long as

$$u^I(p_a) + \tau \geq u^I(p_c) - \theta \iff \tau \geq u^I(p_c) - u^I(p_a) - \theta \equiv \tau_{ac}^I(p_a, p_c). \tag{15}$$

Likewise, group-$I \in \{A, B\}$ voters prefer b over c if:

$$u^I(p_b) - \tau \geq u^I(p_c) - \theta \iff \tau \leq u^I(p_b) - u^I(p_c) + \theta \equiv \tau_{bc}^I(p_b, p_c). \tag{16}$$

---

[8] Our analysis accounts for the fact that our assumption $\rho < u/2$ implies that each group casts a positive fraction of first preferences, but does not ensure that each group casts a positive fraction of second preferences.

Finally, group-C voters prefer a over b if and only if

$$u^C(p_a) + \tau - \theta \geq u^C(p_b) - \tau - \theta \iff \tau \geq \frac{u^C(p_b) - u^C(p_a)}{2} \equiv \tau_{ab}^C(p_a, p_b). \tag{17}$$

Given a profile $p = (p_a, p_b, p_c)$, candidate a's total second preferences are therefore:[9]

$$
\begin{aligned}
v_a^s(p, \tau) \equiv\ & \alpha\phi \max\{0, u^A(p_a) + \tau - \rho\}\mathbb{I}\{\tau_{ac}^A(p_a, p_c) \leq \tau < \tau_{ab}^A(p_a, p_b)\} \\
& + (1-\alpha)\phi \max\{0, u^B(p_a) + \tau - \rho\}\mathbb{I}\{\tau_{ac}^B(p_a, p_c) \leq \tau \leq \tau_{ab}^B(p_a, p_b))\} \\
& + \gamma\phi \max\{0, u^C(p_a) + \tau - \theta - \rho\}\mathbb{I}\{\tau_{ab}^C(p_a, p_b) \leq \tau < \tau_{ac}^C(p_a, p_c)\}.
\end{aligned} \tag{18}
$$

Similarly, b's second preferences are

$$
\begin{aligned}
v_b^s(p, \tau) \equiv\ & \alpha\phi \max\{0, u^A(p_b) - \tau - \rho\}\mathbb{I}\{\tau_{ab}^A(p_a, p_b) \leq \tau \leq \tau_{bc}^A(p_b, p_c)\} \\
& + (1-\alpha)\phi \max\{0, u^B(p_b) - \tau - \rho\}\mathbb{I}\{\tau_{ab}^B(p_a, p_b) < \tau < \tau_{bc}^B(p_b, p_c)\} \\
& + \gamma\phi \max\{0, u^C(p_b) - \tau - \theta - \rho\}\mathbb{I}\{\tau_{bc}^C(p_b, p_c) < \tau \leq \tau_{ab}^C(p_a, p_b)\}.
\end{aligned} \tag{19}
$$

Finally, c's second preferences are

$$
\begin{aligned}
v_c^s(p, \tau) \equiv\ & \alpha\phi \max\{0, u^A(p_c) - \theta - \rho\}\mathbb{I}\{\tau > \tau_{bc}^A(p_b, p_c) \text{ or } \tau < \tau_{ac}^A(p_a, p_c)\} \\
& + (1-\alpha)\phi \max\{0, u^B(p_c) - \theta - \rho\}\mathbb{I}\{\tau > \tau_{bc}^B(p_b, p_c) \text{ or } \tau < \tau_{ac}^B(p_a, p_c)\} \\
& + \gamma\phi \max\{0, u^C(p_c) - \rho\}\mathbb{I}\{\tau > \tau_{ac}^C(p_a, p_c) \text{ or } \tau < \tau_{bc}^C(p_b, p_c)\}.
\end{aligned} \tag{20}
$$

We begin by conjecturing that candidate c targets her base with policy t^C. Recall that under plurality, this was c's dominant strategy: candidate c cannot win the first preference of any voter outside of her base, and therefore has no reason to appeal to majority voters with a broad policy. Under RCV, however, candidate c might want to campaign broadly in the hopes of winning *second* preferences from voters in either majority group A or B. We return to this consideration, below.

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[9] To derive these expressions, recognize (i) that for any (p_a, p_b, p_c), $\tau_{bc}^C(p_b, p_c) < \tau_{ab}^C(p_a, p_b) < \tau_{ac}^C(p_a, p_c)$, and (ii) that Lemma 1 implies for each $I \in \{A, B\}$ and any $p = (p_a, p_b, p_c)$ that $\tau_{bc}^I(p_b, p_c) > \tau_{ab}^I(p_a, p_b) > \tau_{ac}^I(p_a, p_c)$.

When $p_c = t^C$, candidate a's first preferences are still given by expression (4), b's are still given by expression (5), and c's total first preferences are $\gamma\phi(1-\rho)$. The second preferences of minority group C voters therefore play no role, since candidate c either wins the most or second-highest first preferences (Lemma 3). For any pair (p_a, p_b), we can define a critical realization of the aggregate shock that determines whether candidate $j \in \{a, b\}$'s sum of first and second preferences exceed c's total first and second preferences:

$$\tau^{ac}_{\text{rcv}}(p_a, p_b, t^C) \equiv \inf\{\tau : v^f_a(p_a, p_b, t^C, \tau) + \overbrace{v^s_a(p_a, p_b, t^C, \tau)}^{a\text{'s 2}^{\text{nd}}\text{ preferences}} \geq \gamma\phi(1-\rho) + \overbrace{v^s_c(p_a, p_b, t^C, \tau)}^{c\text{'s 2}^{\text{nd}}\text{ preferences}}\} \quad (21)$$

$$\tau^{bc}_{\text{rcv}}(p_a, p_b, t^C) \equiv \sup\{\tau : v^f_b(p_a, p_b, t^C, \tau) + \underbrace{v^s_b(p_a, p_b, t^C, \tau)}_{b\text{'s 2}^{\text{nd}}\text{ preferences}} \leq \gamma\phi(1-\rho) + v^s_c(p_a, p_b, t^C, \tau)\}. \quad (22)$$

A pair of mutual best responses (p^*_a, p^*_b) to c's strategy of $p_c = t^C$ satisfies

$$p^*_a = \arg\min_{p_a} \left\{\max\{\tau^{ac}_{\text{rcv}}(p_a, p^*_b, t^C), \tau^{ab}(p_a, p^*_b)\}\right\} \quad (23)$$

$$p^*_b = \arg\max_{p_b} \left\{\min\{\tau^{bc}_{\text{rcv}}(p^*_a, p_b, t^C), \tau^{ab}(p^*_a, p_b)\}\right\}. \quad (24)$$

In words: candidate a's objective is to weaken the most stringent of two conditions: winning enough first preferences to defeat b and winning enough first *and* second preferences to defeat c. The interpretation of b's objective is similar.

Our analysis proceeds by verifying mutual best responses p^*_a and p^*_b to candidate c's strategy; then, we verify that candidate c's strategy of targeting her base with policy $p_c = t^C$ is a best response to these choices by candidates a and b.

Objectives (23) and (24) embody key facts about RCV:

(1) Winning voters' second preferences may be valuable.

(2) Winning voters' first preferences may not be necessary to benefit from their support.

These facts are used to support the argument that RCV should incentivize a broad campaigning strategy. Our paper's insight is that these facts need *not* incentivize a broad strategy: they may in fact intensify a candidate's incentives to pursue a targeted campaign.

Recall from our earlier analysis that the majority *never* divides on first preferences when

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a and b converge on the broad strategy: $p_a = p_b = g$. Because the majority never divides, their second preferences play no role in the election outcome. Thus, a wins under exactly the same conditions as plurality: if and only if her combined support from the majority exceeds c's total first preferences from minority voters:

$$\phi(u + \tau - \rho) > \gamma\phi(1 - \rho) \iff \tau > \max\{0, \gamma - u + \rho(1 - \gamma)\}.$$

Suppose, however, that candidate a unilaterally reverts to targeting her core supporters with policy $p_a = t^A$, when candidate b pursues a broad strategy $p_b = g$. As in plurality, majority voters divide on first preferences whenever $\tau \in [(u-1)/2, u/2]$, and a's first preferences are given by $v_a^f(t^A, g, t^C\tau)$ in expression (13) and highlighted in Figure 2. When the majority divides, candidate a's first preferences are $\alpha\phi(1 + \tau - \rho)$, while b's are $(1 - \alpha)\phi(u - \tau - \rho)$. Candidate a therefore wins more first preferences than candidate b for whenever she is preferred by group-A voters. The reasons are two-fold: advantaged a's base of size α is larger than b's base of mass $1 - \alpha$, and a's base strategy better-mobilizes her core supporters than b's broad strategy.

So, when $p_a = t^A$, $p_b = g$ and $p_c = t^C$, candidate b wins the fewest first preferences and is eliminated in the first round of vote counting whenever $\tau \geq (u - 1)/2$. What are a's second preferences in that context?[10] Using expression (16), recognize that group-B voters prefer a to c whenever $\tau \geq \tau_{ac}^B(t^A, t^C) = -\theta$. Since $\theta > u/2$ and $u > 1/2$, we therefore have $\tau_{ac}^B(t^A, t^C) < (u-1)/2$. In words: as a consequence of c's choice to target her core supporters, a is assured that group-B voters prefer a to c whenever preference shock τ is favorable enough that a defeats b with first preferences.

Figure 6 shows candidate a's first (thick red) and second (dashed red) preferences at the profile $(p_a, p_b, p_c) = (t^A, g, t^B)$. It further shows candidate a's total first (blue) and second (dashed blue) preferences at the profile (g, g). The Figure highlights that candidate a's total first and second preferences under the strategy (t^A, g, t^C) exceed c's whenever:

---

[10] Our analysis allows for the possibility that a candidate could win a majority of preferences in the first round and thus second preferences play no role in the election's outcomes.

$$\alpha\phi(1+\tau-\rho) + (1-\alpha)\phi\min\{0, \tau-\rho\} > \gamma\phi(1-\rho)$$

$$\Longleftrightarrow \tau > \min\left\{\gamma - \alpha + \rho(1-\gamma), \frac{\gamma-\alpha}{\alpha}(1-\rho)\right\} \equiv \tau_{\mathrm{rcv}}^{ac}(t^A, g, t^C).$$

Notice that, in the example, candidate a maximizes her sum of first and second preferences by pursuing a broad strategy. The intuition is precisely as under plurality: $u > \alpha$ implies that it is easier for a to mobilize turnout amongst a unified majority with a broad campaign than with one that is exclusively targeted to her core supporters.

Nonetheless, a benefits from second preferences under RCV *only if* she wins more first preferences than b. Figure 6 highlights that whenever $\tau \in [(u-1)/2, 0]$, candidate a wins more first preferences than candidate b when she pursues a targeted strategy, but wins fewer first preferences than b when she pursues a broad strategy . So, despite winning fewer total first and second preferences by targeting her base, the second preferences are *more valuable* to a when she differentiates from b than they are when she campaigns broadly.

We conclude that a wins the election under RCV when $p = (t^A, g, t^C)$ if and only if she wins
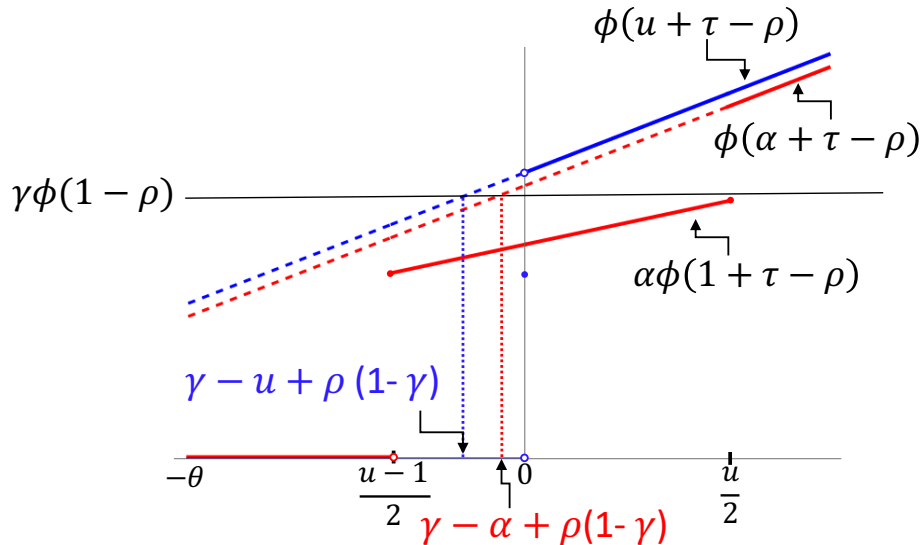

Figure 6 – Majority candidate a's first (thick red) and second preferences (dashed red) at the profile (t^A, g, t^C) and her first (thick blue) and second preferences (dashed blue) at the profile (g, g, t^C). The black line is candidate c's total first and second preferences.

more first preferences than b and also wins more combined first and second preferences than c:

$$\tau > \max\{\tau^{ab}(t^A, g), \tau^{ac}_{\text{rcv}}(t^A, g, t^C)\}$$
$$= \max\left\{\frac{u-1}{2}, \min\left\{\gamma - \alpha + \rho(1-\gamma), \frac{\gamma-\alpha}{\alpha}(1-\rho)\right\}\right\}.$$

Our next proposition shows that RCV *intensifies* a's incentive to revert to her base even in high-apathy settings where plurality yields convergence. It also shows that RCV cannot sustain convergence in *low*-apathy environments. Recall that $\bar{\rho} \equiv 1 - \frac{\alpha}{1-\alpha}\frac{1-u}{\gamma}$ was defined as the threshold degree of apathy such that broad campaigning strategies by the majority can be sustained if and only if $\rho \leq \bar{\rho}$. Define $\underline{\rho} \equiv \frac{\alpha-\gamma}{1-\gamma}$.

Proposition 4. *Under RCV, an equilibrium exists in which both majority candidates offer the broad policy if and only if $\underline{\rho} \leq \rho \leq \bar{\rho}$. In this equilibrium, candidate c targets her core supporters.*

Recall that under plurality, broad campaigning strategies by the majority could not be sustained in relatively high-apathy environments, i.e., $\rho \geq \bar{\rho}$. The reason was that both majority candidates gambled on winning enough first preferences solely from their core supporters to defeat c outright. The possibility of winning second preferences under RCV does not impact this incentive, and thus RCV cannot abate the candidates' incentives in these contexts.

The proposition also states that broad convergence fails in *low*-apathy contexts where it can be sustained under plurality. Why? Recall that higher apathy reduces average participation, and therefore reduces group size advantages. Conversely, lower apathy—i.e., lower ρ—generates higher baseline participation across all groups. This

(1) increases the majority's size advantage over the minority of $1 - \gamma > 0$, and

(2) increases the minority's size advantage over a's core supporters of $\gamma - \alpha > 0$.

Under plurality, both effects encourage a to pursue a united majority with a broad strategy, instead of focusing exclusively on her core supporters. Higher baseline turnout has one final consequence that is only relevant under RCV: it

(3) increases the size advantage of a's base over b's base of $\alpha - (1 - \alpha) > 0$.

Recall that defeating b yields no direct benefit under plurality, since candidate a's relevant

pairwise contest is with c (Lemma 3). Under RCV, by contrast, defeating b generates a second preference dividend, which effectively lends a the benefit of a united majority—i.e., the benefit described in point (1), above. This benefit accrues in spite of the fact that the majority's first preferences divide between candidates a and b. It accrues solely under RCV, and not plurality.

We conclude that larger baseline participation and engagement discourage a base strategy under plurality, but *encourages* it under RCV.

These observations underlie the comparison of plurality versus RCV that is the main message of our paper. Under plurality rule, candidates who tend to appeal to similar groups in the electorate draw votes from one another when their voters disagree about who to rank first. RCV makes vote-splitting less costly by allowing voters to express more preferences. The ability to express more preferences implicitly solves the voters' coordination problem that exists if the majority divides. Nonetheless, the vote-splitting problem only arises when the majority divides, and the majority is prone to divide when the candidates offer distinct policies. Since RCV's benefit to candidates accrues when their voters divide, it provides comparatively less discipline against electoral strategies that generate division.

Simply because second preferences are valuable does not imply that candidates choose policies to maximize them. Second preferences are only valuable from voters who ranked some other candidate first, and whose most-preferred candidate has already been eliminated from the contest.

Figure 6 and the accompanying discussion starkly illustrate this point. They show how RCV changes a candidate's value from winning *first* preferences. Under plurality, she needs more first preferences than *every other candidate*. Under RCV, her an alternative path to victory may be to securing more first preferences than other candidates *whose voters are likely to rank her second*. When this path calls on a candidate to differentiate from those candidates, we may observe policy divergence under RCV in contexts where plurality guarantees convergence.

Corollary 1. *If broad strategies by candidates a and b can be supported in an equilibrium under RCV, then they can also be supported in an equilibrium under plurality. The reverse is not true.*

What else can happen under RCV? The answer depends on the degree of majority-minority mis-alignment, captured by the parameter θ.
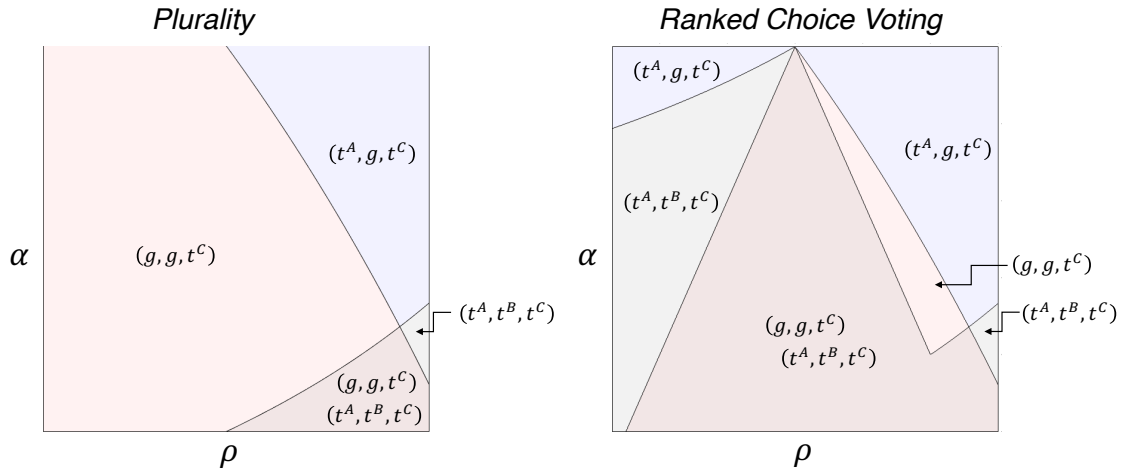
Figure 7 – Comparing equilibria under plurality and RCV for benchmark parameters for *high polarization*, $\theta > \frac{u+1}{2}$. The left-hand panel corresponds to plurality rule, and replicates Figure 5. The right-hand panel shows the corresponding equilibrium set under RCV. *Parameters: $u = .6$ and $\gamma = .6$.*

Large polarization. When $\theta > (u + 1)/2$, advantaged majority candidate a always targets her base in sufficiently low- or high-apathy contexts, i.e., if $\rho < \underline{\rho}$ or $\rho > \overline{\rho}$. As under plurality rule, candidate b's incentives depend on the extent of her own core supporters' size disadvantage relative to a's core supporters. When a's core supporters are a large share $\alpha > .5$ of the majority, b's best path to victory remains a broad strategy that maximizes her chances of uniting and turning out the majority. If, instead, α is close enough to one half, b's responds to a's base strategy in kind. Under RCV, b's incentive to revert to her base is again stronger than plurality, since she wins support from group-A voters even without securing their first preferences.

Proposition 5. *When polarization is large, i.e., $\theta \geq (u + 1)/2$, there exists $\alpha^{rcv} \geq \alpha^{plu}$ such that:*

(1) if and only if $\alpha > \alpha^{rcv}$ and either $\rho < \underline{\rho}$ or $\rho > \overline{\rho}$, (t^A, g, t^C) is an equilibrium and

(2) whenever $\alpha \leq \alpha^{rcv}$, (t^A, t^B, t^C) is an equilibrium.

Figure 7 illustrates the equilibria for different pairs (ρ, α) under plurality (left-hand panel) versus RCV (right-hand panel). Note that in large polarization contexts, Propositions 4 and 5 characterize the complete set of pure strategy equilibria (imposing that c does not play a weakly dominated strategy), and a unique pure strategy equilibrium is also the unique Nash equilibrium.

Since $\alpha^{\text{rcv}} \geq \alpha^{\text{plu}}$ (with strict inequality for some primitives), we obtain a partial converse to Corollary 1.

Corollary 2. *If polarization is large: when plurality rule supports an equilibrium in which every candidate targets her base, then so too does RCV, but the reverse is not true.*

Figure 7 also highlights that multiplicity can arise under qualitatively similar circumstances to plurality. This multiplicity again reflects a coordination trap: each candidate a's and b's probability of winning is strictly lower in the (t^A, t^B, t^C) equilibrium than in the (g, g, t^C) equilibrium, and voters are also (on average) strictly worse off. In the Figure, RCV *both* exacerbates the candidates' coordination trap relative to plurality *and* increases the circumstances in which every candidate targets their base in the unique equilibrium.

Low Polarization. The significance of large polarization $\theta \geq (u + 1)/2$ in our previous result lies exclusively with candidate c's incentives. When c targets her base, she foregoes any prospect of winning second preferences from majority voters, regardless of the majority candidates' strategies. Candidate c's targeted campaign therefore exerts no discipline on the strategies of the majority candidates a and b. High polarization further implies that c fails to win second preferences from majority voters even when she pursues a broad campaign. This implies that her dominant strategy is to target her base. With lower polarization (i.e., θ small) candidate c can win second preferences from some majority voters if she reverts to a broad campaign. Nonetheless, our previous results extend in high-apathy contexts.

Proposition 6. *For all $\theta > u/2$, there exists $\alpha^{\text{rcv}} \geq \alpha^{\text{plu}}$ such that if $\rho > \overline{\rho}^{\text{rcv}}$ and $\alpha \leq \alpha^{\text{rcv}}$, (t^A, t^B, t^C) is an equilibrium, and if $\alpha \geq \alpha^{\text{rcv}}$ (t^A, g, t^C) is an equilibrium. Thus, in high-apathy environments, when plurality rule supports an equilibrium in which every candidate targets her base, so too does RCV, but the reverse is not true.*

To understand why, suppose candidates a and b target their bases with policies t^A and t^B, respectively. By pursuing a broad campaign, candidate c wins second preferences from voters in the majority groups A (if $\tau \geq \theta - u$) and B (if $\tau \leq u - \theta$). These second preferences convey no electoral benefit under plurality, but under RCV they may tilt the election in c's favor simply by denying those second preferences to either candidates a or b. Why, then, doesn't

c pursue them? If c reverts from targeting her base to a broad campaigning strategy she sacrifices turnout from her core supporters. In high-apathy contexts, mobilization is the critical margin: candidate c's loss in turnout from her own base more than offsets her benefit from any second preferences she receives from majority voters.

Together, Propositions 5 and 6 establish that in political environments characterized *either* by relatively high polarization, *or* by relatively large apathy and therefore low participation, RCV always encourages candidate to target their bases to a greater extent than under plurality.

Consider, finally, contexts characterized by both low polarization and relatively high levels of average participation, i.e., θ small and $\rho < \bar{\rho}$. Plurality always sustains an equilibrium in which the majority candidates campaign broadly (Proposition 1), but may also support an equilibrium in which every candidate targets her base when α is small enough (Proposition 2). So, these are contexts in which candidates under plurality may face a coordination problem. In the Appendix, we show that RCV may *not* support an equilibrium in which every candidate targets her base. The reason is that in high-participation and low-polarization contexts, c may find it valuable to court second preferences from majority voters by way of a broad strategy. So, while RCV still cannot induce the majority candidates to campaign broadly in situations where plurality also cannot (that is, Corollary 1 extends), it may not lead every candidate to target her base when plurality does—that is, Corollary 2 may not extend.

5. Preference Alignment

We now ask whether RCV better necessarily aligns electoral outcomes with voters' preferences to a greater extent than plurality.

Recall that a *Condorcet Winner* is a candidate that defeats every other candidate in pairwise majority voting. We cannot define an ex-ante Condorcet Winner in our framework because that candidate's identity depends on her (endogenous) campaigning strategy. However, we *can* define a related concept of a *Condorcet Loser*. A Condorcet loser is a candidate that is defeated by any other candidate in pairwise majority voting. By definition, a Condorcet loser cannot win a two-candidate contest, but may prevail in a multi-candidate election due to vote-splitting.

In our framework, candidate c is an ex-ante Condorcet loser so long as θ is sufficiently large: in particular, when $\theta > 1$ every voter in the majority likes candidate c the least regardless of the preference shock τ's realization, and regardless of the candidates' campaigning strategies. Recall that for some parameters we may have multiple equilibria under both systems. Focusing on primitives for which there is a unique equilibrium under both systems, the next result identifies conditions under which RCV *increases* a Condorcet loser's probability of winning the election, relative to plurality.

Proposition 7. *Suppose $\theta > 1$, so that candidate c is a Condorcet Loser. Then, c's probability of winning under is* lower *under RCV than plurality if $\rho > \bar{\rho}$, but* higher *under RCV than plurality if $\rho < \underline{\rho}$ and $\alpha > \alpha^{plu}$.*

Depending on parameters, the orderings may be weak or strict. Recognize that for fixed strategies, RCV *always* (weakly) improves the probability that a majority candidate wins. Remarkably, Proposition 7 shows that the strategies pursued by the majority candidates may distort outcomes to such a degree that c's victory prospects *increase*. This occurs when the unique equilibrium under plurality leads the majority candidates to pursue a broad strategy, whereas the candidates instead target their bases under RCV's unique equilibrium.

Regardless of whether c's prospects improve or weaken under RCV, the next result shows that candidate a with the relatively larger base of cores supporters is *always* better off under RCV, but that weaker b's prospects may diminish under RCV relative to plurality. We continue to focus on the same parameters as the previous proposition.

Proposition 8. *Suppose $\theta > 1$.*

(1) If $\rho > \bar{\rho}$, both a's and b's probabilities of winning are both higher *under RCV.*

(2) If $\rho < \underline{\rho}$ and $\alpha > \alpha^{plu}$, a's probability of winning is strictly higher under RCV, but b's probability of winning is strictly lower *under RCV.*

In particular, for primitives such that the Condorcet loser's victory prospects increase under RCV, that electoral improvement is exclusively at the cost of the candidate representing the smaller group within the majority. While scholars argue that RCV improves the electoral

prospects of candidates representing electoral minorities (e.g., Benade et al., 2021), the propositions unearths circumstances in which the *opposite* is true.

Although welfare comparisons are hampered by equilibrium multiplicity, some broad observations are possible. RCV *always* generates weakly higher welfare than plurality in high-apathy contexts ($\rho > \overline{\rho}$), though depending on other primitives the systems may generate the same welfare.[11] For instance, under the benchmark parameters in Figure 7 plurality and RCV yield identical welfare for all $\rho > \overline{\rho}$. For intermediate levels of voter apathy $\underline{\rho} < \rho < \overline{\rho}$, both systems can support broad convergence by the majority either as a unique equilibrium or (depending on parameters) in conjunction with other equilibria. When both systems support an equilibrium in which every candidate targets her base, RCV dominates plurality if that equilibrium is played under each system, but this equilibrium is strictly dominated by the convergent equilibrium (g, g, t^C), which may be unique under plurality even when RCV supports both (as in Figure 7 when $\alpha^{\text{plu}} < \alpha < \alpha^{\text{rcv}}$). Finally, when $\rho < \underline{\rho}$, only plurality supports an equilibrium in which the majority candidates campaign broadly, and this equilibrium generates strictly higher welfare than RCV for all levels of polarization satisfying Assumption 2. Both systems may nonetheless support multiple equilibria for these primitives.

6. Conclusion

Our paper studies electoral competition under Ranked Choice Voting (RCV). We ask: does RCV necessarily provide greater incentives for candidates to moderate their policy platforms than plurality rule? Does RCV better-align electoral outcomes with voters' preferences?

Our framework yields the following results. When RCV encourages broad campaigning strategies, so does plurality, but the reverse is not true; and, when plurality encourages candidates to target their bases, so does RCV in contexts either of polarization *or* low baseline participation. In those environments, RCV therefore fails to moderate candidates' campaigning strategies. When a Condorcet Loser exists, RCV may weaken her electoral prospects for some primitives, but we unearth contexts in which the candidates' strategic choices offset RCV's

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[11] Specifically, when $\rho > \overline{\rho}$: the systems yield the same welfare for $\alpha \geq \alpha^{\text{rcv}}$, RCV yields strictly higher welfare if $\alpha \in (\alpha^{\text{plu}}, \alpha^{\text{rcv}})$, and when $\alpha \leq \alpha^{\text{plu}}$ RCV either yields the same welfare or strictly higher than plurality, depending further on primitives.

benefits from second preferences to such a degree that a Condorcet loser's victory prospects may *increase*, relative to plurality.

We close with a broader interpretation of our results, and how they relate to existing arguments that favor RCV's adoption. By allowing voters to express a preference for multiple candidates, RCV implicitly helps voters to solve a coordination problem they would otherwise face in multi-candidate elections under plurality rule. For a fixed set of alternatives, this improved implicit coordination facilitates the election of moderate policies, and in particular majority-preferred policies when they exist. However, this improved implicit coordination also changes the candidates' strategies, by opening up new pathways to electoral victory that may be absent under plurality. Changes in electoral rules therefore have the potential to create new conflicts between candidates whose consequences can be difficult to predict. Indeed, those consequences may be opposite to the aspirations of both scholars and reformers of electoral systems.

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7. Appendix: Proofs

Notation. For majority group $I \in \{A, B, C\}$, define thresholds

$$\tau_{ab}^{I}(p_a, p_b) \equiv \frac{u^I(p_b) - u^I(p_a)}{2} \tag{25}$$

$$\tau_{ac}^{I}(p_a, p_c) \equiv \begin{cases} u^I(p_c) - u^I(p_a) - \theta & \text{if } I \in \{A, B\} \\ u^I(p_c) - u^I(p_a) + \theta & \text{if } I = C \end{cases} \tag{26}$$

$$\tau_{bc}^{I}(p_b, p_c) \equiv \begin{cases} u^I(p_b) - u^I(p_c) + \theta & \text{if } I \in \{A, B\} \\ u^I(p_b) - u^I(p_c) - \theta & \text{if } I = C. \end{cases} \tag{27}$$

For candidate $j \in \{b, c\}$, threshold $\tau_{aj}^{I}(p_a, p_j)$ is the critical aggregate shock realization such that group-I voters prefer a to j if and only if $\tau \geq \tau_{aj}^{I}(p_a, p_j)$. Similarly, $\tau_{bc}^{I}(p_b, p_c)$ is the critical aggregate shock realization such that group-I voters prefer b to c if and only if $\tau \leq \tau_{bc}^{I}(p_a, p_c)$.

For any $z \in \mathbb{R}$, we let $F(z)$ denote the cumulative distribution of the reservation value η, uniformly distributed on the interval $[\rho, \rho + \phi^{-1}]$, evaluated at z; i.e.,

$$F(z) \equiv \max\{\phi(z - \rho), 0\}.$$

Our assumption that $\phi < \frac{2}{3-2\rho}$ implies that $F(U_j^I) < 1$ for all candidates $j \in \{a, b, c\}$, and policy profile $p = (p_a, p_b, p_c)$ and groups $I \in \{A, B, C\}$.

Proof of Lemma 1. The result is true if for each $I \in \{A, B\}$ and any $p = (p_a, p_b, p_c)$, $\tau_{bc}^{I}(p_b, p_c) > \tau_{ab}^{I}(p_a, p_b) > \tau_{ac}^{I}(p_a, p_c)$. We have $\tau_{bc}^{I}(p_b, p_c) > \tau_{ab}^{I}(p_a, p_b)$ if $u^I(p_b) - u^I(p_c) + \theta > \frac{u^I(p_b) - u^I(p_a)}{2}$, which holds for any $p = (p_a, p_b, p_c)$ if and only if $\theta > u/2$. The argument that $\tau_{ab}^{I}(p_a, p_b) > \tau_{ac}^{I}(p_a, p_c)$ for any p is similar. $\square$

Proof of Lemma 2. The previous lemma implies that group-$I \in \{A, B\}$ voters never cast first preferences for candidate c. A group-C voter prefers candidate c to a if and only if $\tau \leq \tau_{ac}^{C}(p_a, p_c)$, and prefers c to b if and only if $\tau \geq \tau_{bc}^{C}(p_b, p_c) = u^C(p_b) - u^C(p_c) - \theta$. It is easily verified that for any (p_a, p_b, p_c), $\tau_{ac}^{C}(p_a, p_c) > \tau_{ab}^{B}(p_a, p_b)$ and $\tau_{bc}^{C}(p_b, p_c) < \tau_{ab}^{A}(p_a, p_b)$. So,

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candidate a's total first preferences for any $p = (p_a, p_b, p_c)$ are

$$
v_a^f(p, \tau) = \begin{cases}
0 & \text{if } \tau < \tau_{ab}^A(p_a, p_b) \\
\alpha F(u^A(p_a) + \tau) & \text{if } \tau \in [\tau_{ab}^A(p_a, p_b), \tau_{ab}^B(p_a, p_b)] \\
\alpha F(u^A(p_a) + \tau) + (1 - \alpha)F(u^B(p_a) + \tau) & \text{if } \tau \in (\tau_{ab}^B(p_a, p_b), \min\{1/2, \tau_{ac}^C(p_a, p_c)\}] \\
\alpha F(u^A(p_a) + \tau) + (1 - \alpha)F(u^B(p_a) + \tau) & \\
\quad + \gamma F(u^C(p_a) + \tau - \theta) & \text{if } \tau \in (\min\{1/2, \tau_{ac}^C(p_a, p_c)\}, 1/2].
\end{cases}
$$
(28)

Similar steps yield b's first preferences:

$$
v_b^f(p, \tau) = \begin{cases}
0 & \text{if } \tau > \tau_{ab}^B(p_a, p_b) \\
(1 - \alpha)F(u^B(p_b) - \tau) & \text{if } \tau \in [\tau_{ab}^A(p_a, p_b), \tau_{ab}^B(p_a, p_b)] \\
\alpha F(u^A(p_b) - \tau) + (1 - \alpha)F(u^B(p_b) - \tau) & \text{if } \tau \in [\max\{-1/2, \tau_{bc}^C(p_b, p_c)\}, \tau_{ab}^A(p_a, p_b)) \\
\alpha F(u^A(p_b) - \tau) + (1 - \alpha)F(u^B(p_b) - \tau) & \\
\quad + \gamma F(u^C(p_b) - \tau - \theta) & \text{if } \tau \in [-1/2, \max\{-1/2, \tau_{bc}^C(p_b, p_c)\}).
\end{cases}
$$
(29)

Finally, candidate c's first preferences are

$$
v_c^f(p, \tau) = \begin{cases}
0 & \text{if } \tau \in [-1/2, \max\{-1/2, \tau_{bc}^C(p_b, p_c)\}) \\
\gamma F(u^C(p_c)) & \text{if } \tau \in [\max\{-1/2, \tau_{bc}^C(p_b, p_c)\}, \min\{1/2, \tau_{ac}^C(p_a, p_c)\}] \\
0 & \text{if } \tau \in (\min\{1/2, \tau_{ac}^C(p_a, p_c)\}, 1/2].
\end{cases}
$$
(30)

Notice that when $p_c = t^C$, (28) and (29) specialize to expressions (4) and (5) in the text, respectively, since for either $p_a \in \{g, t^A\}$, $\tau_{ac}^C(p_a, t^C) \geq 1 - u + \theta > 1 - \frac{u}{2} > \frac{1}{2}$, and likewise for either $p_b \in \{g, t^B\}$, $\tau_{bc}^C(p_b, t^C) < -\frac{1}{2}$. This further implies that for all $\tau \in \left[-\frac{1}{2}, -\frac{1}{2}\right]$, $v_c^f(p_a, p_b, t^C) = \gamma F(1) = \gamma \phi(1 - \rho)$, as claimed in the text. We define

$$
\tau_{\text{plu}}^{ac}(p_a, p_b, p_c) \equiv \inf\{\tau : v_a^f(p, \tau) \geq v_c^f(p, \tau)\}
$$
$$
\tau_{\text{plu}}^{bc}(p_a, p_b, p_c) \equiv \sup\{\tau : v_b^f(p, \tau) \leq v_c^f(p, \tau)\}
$$
$$
\tau_{\text{plu}}^{ab}(p_a, p_b, p_c) \equiv \inf\{\tau : v_a^f(p, \tau) \geq v_b^f(p, \tau)\}.
$$

For any pair (p_a, p_b), $\tau_{\text{plu}}^{ac}(p_a, p_b, t^C) \geq \tau_{\text{plu}}^{ac}(p_a, p_b, g)$ and $\tau_{\text{plu}}^{bc}(p_a, p_b, t^C) \leq \tau_{\text{plu}}^{bc}(p_a, p_b, g)$, where these thresholds are defined in (6) and (7). This yields that $p_c = t^C$ (weakly) maximizes c's probability of winning for any pair of actions $p_a \in \{g, t^A\}$ and $p_b \in \{g, t^B\}$.

For the benefit of later results, we strengthen this lemma by showing that if $\rho > \frac{u-\gamma}{1-\gamma}$, t^C is c's weakly dominant strategy. To do so, we need only verify that there exists a pair $(p_a, p_b) \in \{g, t^A\} \times \{g, t^B\}$ such that either $\tau_{\text{plu}}^{ac}(p_a, p_b, t^C) > \tau_{\text{plu}}^{ac}(p_a, p_b, g)$ or $\tau_{\text{plu}}^{bc}(p_a, p_b, t^C) < \tau_{\text{plu}}^{bc}(p_a, p_b, g)$. This follows from the fact that if $\rho > \frac{u-\gamma}{1-\gamma}$, then:

$$\tau_{\text{plu}}^{ac}(g, g, t^C) = \max\{0, \gamma - u + \rho(1 - \gamma)\} = \gamma - u + \rho(1 - \gamma) = -\tau_{\text{plu}}^{bc}(g, g, t^C) > 0,$$

whereas, for any primitives:

$$\tau_{\text{plu}}^{ac}(g, g, g) = \max\{0, \gamma u - u - \rho(1 - \gamma)\} = -\tau_{\text{plu}}^{bc}(g, g, g) = 0. \ \square$$

Proof of Lemma 3. We prove the argument for candidate $j = b$, since the argument for candidate a is similar. Using (28) and (29), we obtain candidate a's total first preferences:

$$v_a^f(p_a, p_b, t^C, \tau) = \begin{cases} 0 & \text{if } \tau < \tau_{ab}^A(p_a, p_b) \\ \alpha F(u^A(p_a) + \tau) & \text{if } \tau \in [\tau_{ab}^A(p_a, p_b), \tau_{ab}^B(p_a, p_b)] \\ \alpha F(u^A(p_a) + \tau) + (1 - \alpha)F(u^B(p_a) + \tau) & \text{if } \tau > \tau_{ab}^B(p_a, p_b). \end{cases}$$
(31)

and candidate b's total first preferences:

$$v_b^f(p_a, p_b, t^C, \tau) = \begin{cases} 0 & \text{if } \tau > \tau_{ab}^B(p_a, p_b) \\ (1 - \alpha)F(u^B(p_b) - \tau) & \text{if } \tau \in [\tau_{ab}^A(p_a, p_b), \tau_{ab}^B(p_a, p_b)] \\ \alpha F(u^A(p_b) - \tau) + (1 - \alpha)F(u^B(p_b) - \tau) & \text{if } \tau < \tau_{ab}^A(p_a, p_b). \end{cases}$$
(32)

Finally, c's total first preferences for all $\tau \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ are $\gamma F(1)$. Notice that $v_b^f(p_a, p_b, t^C, \tau) \geq \gamma F(1)$ requires that either

$$\tau \in [\tau_{ab}^A(p_a, p_b), \tau_{ab}^B(p_a, p_b)] \quad \text{and} \quad (1 - \alpha)F(u^B(p_b) - \tau) \geq \gamma F(1),$$

or, instead, that

$$\tau < \tau_{ab}^A(p_a, p_b) \quad \text{and} \quad \alpha F(u^A(p_b) - \tau) + (1 - \alpha)F(u^B(p_b) - \tau) \geq \gamma F(1).$$

In the second case, a wins zero first preferences, and the lemma is trivial. In the first case, $u^B(p_b) \leq 1$ implies $\tau < 0$, which implies

$$v_a^f(p_a, p_b, t^C, \tau) = \alpha F(u^A(p_a) + \tau) < \alpha F(1) < \gamma F(1) \leq (1 - \alpha)F(u^B(p_b) - \tau) = v_b^f(p_a, p_b, t^C, \tau). \quad \square$$

Proof of Proposition 1. We begin by defining thresholds that will be relevant for our analysis:

$$\tau_1^{ac}(p_a) \equiv \frac{\gamma}{\alpha}(1 - \rho) - u^A(p_a) + \rho \tag{33}$$

$$-\tau_1^{bc}(p_b) \equiv \frac{\gamma}{1 - \alpha}(1 - \rho) - u^B(p_b) + \rho \tag{34}$$

$$\tau_2^{ac}(p_a) \equiv \gamma(1 - \rho) - \alpha u^A(p_a) - (1 - \alpha)u^B(p_a) + \rho \tag{35}$$

$$-\tau_2^{bc}(p_b) \equiv \gamma(1 - \rho) - \alpha u^A(p_b) - (1 - \alpha)u^B(p_b) + \rho. \tag{36}$$

Recalling the following critical thresholds:

$$\tau_{\text{plu}}^{ac}(p_a, p_b, t^C) \equiv \inf\{\tau : v_a^f(p_a, p_b, t^C, \tau) \geq \gamma F(1)\}$$

$$\tau_{\text{plu}}^{bc}(p_a, p_b, t^C) \equiv \sup\{\tau : v_b^f(p_a, p_b, t^C, \tau) \leq \gamma F(1)\},$$

it is easy to verify that

$$\tau_{\text{plu}}^{ac}(p_a, p_b, t^C) = \begin{cases} \tau_2^{ac}(p_a) & \tau_{ab}^B(p_a, p_b) < \tau_2^{ac}(p_a) \\ \tau_{ab}^B(p_a, p_b) & \tau^B(p_a, p_b) \in [\tau_2^{ac}(p_a), \tau_1^{ac}(p_a)) \\ \tau_1^{ac}(p_a) & \tau^B(p_a, p_b) \geq \tau_1^{ac}(p_a), \end{cases}$$

and

$$-\tau_{\text{plu}}^{bc}(p_a, p_b, t^C) = \begin{cases} -\tau_2^{bc}(p_b) & -\tau_{ab}^A(p_a, p_b) < -\tau_2^{bc}(p_a) \\ -\tau_{ab}^A(p_a, p_b) & -\tau_{ab}^A(p_a, p_b) \in [-\tau_2^{bc}(p_a), -\tau_1^{bc}(p_a)) \\ -\tau_1^{bc}(p_b) & \tau_{ab}^A(p_a, p_b) \geq -\tau_1^{bc}(p_a). \end{cases}$$

Notice that

$$-\tau_1^{bc}(g) > \tau_1^{ac}(g) > \frac{1-u}{2} = \tau_{ab}^B(g, t^B) - \tau_{ab}^A(t^A, g) > 0 = \tau_{ab}^B(g, g) = -\tau_{ab}^A(g, g),$$

which implies that

$$\tau_{\text{plu}}^{ac}(g, g, t^C) = \max\{0, \rho(1-\gamma) + \gamma - u\} = -\tau_{\text{plu}}^{bc}(g, g, t^C)$$

$$\tau_{\text{plu}}^{ac}(g, t^B, t^C) = \max\left\{\frac{1-u}{2}, \rho(1-\gamma) + \gamma - u\right\} = -\tau_{\text{plu}}^{bc}(t^A, g, t^C).$$

Further observe that $\gamma \leq 1$ implies

$$\tau_2^{ac}(t^A) - \tau_{ab}^B(t^A, t^B) = \gamma(1-\rho) + \rho - \alpha - \frac{1}{2} < \frac{1}{2} - \alpha < 0,$$

which implies

$$\tau_{\text{plu}}^{ac}(t^A, t^B, t^C) = \min\left\{\frac{1}{2}, \tau_1^{ac}(t^A)\right\}.$$

Finally,

$$-\tau_{\text{plu}}^{bc}(t^A, t^B, t^C) = \min\left\{-\tau_1^{bc}(t^B), \max\left\{-\tau_2^{bc}(t^B), \frac{1}{2}\right\}\right\}$$

$$\tau_{\text{plu}}^{ac}(t^A, g, t^C) = \min\left\{\tau_1^{ac}(t^A), \max\left\{\tau_2^{ac}(t^A), \frac{u}{2}\right\}\right\}$$

$$-\tau_{\text{plu}}^{bc}(g, t^B, t^C) = \min\left\{-\tau_1^{bc}(t^B), \max\left\{-\tau_2^{bc}(t^B), \frac{u}{2}\right\}\right\}$$

with

$$-\tau_{\text{plu}}^{bc}(t^A, t^B, t^C) \geq -\tau_{\text{plu}}^{bc}(g, t^B, t^C) \geq \tau_{\text{plu}}^{ac}(t^A, g, t^C).$$

Recall from Lemma 2 that $p_c = t^C$ is a best response to $p_a = p_b = g$. We therefore derive

conditions under which $p_a = p_b = g$ are mutual best responses to $p_c = t^C$. The conditions are:

$$\begin{cases} \tau_{\text{plu}}^{ac}(g, g, t^C) \leq \tau_{\text{plu}}^{ac}(t^A, g, t^C) \\ -\tau_{\text{plu}}^{bc}(g, g, t^C) \leq -\tau_{\text{plu}}^{bc}(g, t^B, t^C). \end{cases} \tag{37}$$

Since $\tau_{\text{plu}}^{ac}(g, g, t^C) = -\tau_{\text{plu}}^{bc}(g, g, t^C)$ and $\tau_{\text{plu}}^{ac}(t^A, g, t^C) \leq -\tau_{\text{plu}}^{bc}(g, t^B, t^C)$, the conditions in (37) are satisfied if and only if $\tau_{\text{plu}}^{ac}(g, g, t^C) \leq \tau_{\text{plu}}^{ac}(t^A, g, t^C)$, which is true if and only if

$$\max\{0, \rho(1 - \gamma) + \gamma - u\} \leq \min\left\{\tau_1^{ac}(t^A), \max\left\{\tau_2^{ac}(t^A), \frac{u}{2}\right\}\right\}, \tag{38}$$

which holds if and only if

$$\rho(1 - \gamma) + \gamma - u \leq \min\left\{\tau_1^{ac}(t^A), \max\left\{\tau_2^{ac}(t^A), \frac{u}{2}\right\}\right\}.$$

Notice that $u > \alpha$ implies

$$\rho(1 - \gamma) + \gamma - u < \rho(1 - \gamma) + \gamma - u = \tau_2^{ac}(t^A) \leq \max\left\{\tau_2^{ac}(t^A), \frac{u}{2}\right\},$$

and thus (38) holds if and only if

$$\tau_1^{ac}(t^A) \geq \min\left\{\rho(1 - \gamma) + \gamma - u, \frac{u}{2}\right\} \iff \rho \leq \bar{\rho}. \quad \square$$

Proof of Proposition 2. Recall from Lemma 2 that $p_c = t^C$ is a best response to any strategy of candidates a and b. We first verify that (t^A, t^B) are mutual best responses to $p_c = t^C$ if and only if $\alpha \leq \alpha^{\text{plu}}$, where

$$\alpha^{\text{plu}} \equiv 1 - \min\left\{\frac{\gamma(1 - \rho)}{(1 - u)/2 + 1 - \rho}, \frac{\gamma(1 - \rho)}{1 - u + \gamma(1 - \rho)}\right\}.$$

The pair (t^A, t^B) are mutual best responses to $p_c = t^C$ if and only if

$$\begin{cases} \tau_{\text{plu}}^{ac}(t^A, t^B, t^C) \leq \tau_{\text{plu}}^{ac}(g, t^B, t^C) \\ -\tau_{\text{plu}}^{bc}(t^A, t^B, t^C) \leq -\tau_{\text{plu}}^{bc}(t^A, g, t^C), \end{cases}$$

which is equivalent to

$$\begin{cases} \min\left\{\frac{1}{2}, \tau_1^{ac}(t^A)\right\} \leq \max\left\{\frac{1-u}{2}, \rho(1-\gamma)+\gamma-u\right\} \\ \min\left\{-\tau_1^{bc}(t^B), \max\left\{-\tau_2^{bc}(t^B), \frac{1}{2}\right\}\right\} \leq \max\left\{\frac{1-u}{2}, \rho(1-\gamma)+\gamma-u\right\}. \end{cases} \tag{39}$$

Since $-\tau_1^{bc}(t^B) > \tau_1^{ac}(t^A)$, the second condition in (39) is necessary and sufficient. Notice that since $\rho \leq \frac{u}{2}$:

$$\rho(1-\gamma)+\gamma-u \leq \frac{u}{2}(1-\gamma)+\gamma-u < 1-u < \frac{1}{2},$$

which implies that (39) holds if and only if

$$-\tau_1^{bc}(t^B) = \left(\frac{\gamma}{1-\alpha}-1\right)(1-\rho) \leq \max\left\{\frac{1-u}{2}, \rho(1-\gamma)+\gamma-u\right\} \Leftrightarrow \alpha \leq \alpha^{\text{plu}}.$$

Next, we show that (t^A, g) is a pair of mutual best responses to $p_c = t^C$ if and only if $\rho \geq \bar{\rho}$ and $\alpha \geq \alpha^{\text{plu}}$. Pair (t^A, g) are mutual best responses to $p_c = t^C$ if and only if

$$\begin{cases} \tau_{\text{plu}}^{ac}(t^A, g, t^C) \leq \tau_{\text{plu}}^{ac}(g, g, t^C) \\ -\tau_{\text{plu}}^{bc}(t^A, g, t^C) \leq -\tau_{\text{plu}}^{bc}(t^A, t^B, t^C). \end{cases}$$

We already showed that $p_b = g$ is a best response to $p_a = t^A$ and $p_c = t^C$ if and only if $\alpha \geq \alpha^{\text{plu}}$. The argument is completed by recalling from Proposition 1 that $p_a = t^A$ is a best response to $p_b = g$ if and only if $\rho \geq \bar{\rho}$. $\square$

Proof of Remark 1. First, we show that the strategy profile (g, t^B) is never a pair of mutual best responses to $p_c = t^C$. To see why, observe that (g, t^B) are mutual best responses to $p_c = t^C$ if and only if

$$\begin{cases} \tau_{\text{plu}}^{ac}(g, t^B, t^C) \leq \tau_{\text{plu}}^{ac}(t^A, t^B, t^C) \\ -\tau_{\text{plu}}^{bc}(g, t^B, t^C) \leq -\tau_{\text{plu}}^{bc}(g, g, t^C), \end{cases}$$

which is equivalent to

$$\begin{cases} \max\left\{\frac{1-u}{2}, \rho(1-\gamma)+\gamma-u\right\} \leq \min\left\{\frac{1}{2}, \tau_1^{ac}(t^A)\right\} \\ -\tau_1^{bc}(t^B) \leq \rho(1-\gamma)+\gamma-u. \end{cases}$$

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These inequalities imply $\rho(1 - \gamma) + \gamma - u \in [-\tau_1^{bc}(t^B), \tau_1^{ac}(t^A)]$, which is impossible since we already showed $-\tau_1^{bc}(t^B) > \tau_1^{ac}(t^A)$.

To conclude the Remark's proof, we verify two claims. First, if there exists an equilibrium in which c plays a weakly undominated strategy and chooses $p_c = t^C$ with positive probability, then that equilibrium is outcome-equivalent to a pure strategy equilibrium in which $p_a = p_b = g$ and $p_C = t^C$, exists only if $\rho \leq \frac{u - \gamma}{1 - \gamma} < \overline{\rho}$, and is therefore outcome-equivalent to the equilibrium characterized in Proposition 1. Second, a unique pure strategy equilibrium is the unique Nash equilibrium. In other words, when there is a unique pure strategy equilibrium, there is no equilibrium in which at least one candidate randomizes over her actions.

Claim 1. *In any Nash equilibrium in which candidate c does not play a weakly dominated strategy, either c chooses $p_c = t^C$ with probability one, or the equilibrium is outcome-equivalent to the pure strategy equilibrium (g, g, t^C) characterized in Proposition 1.*

Proof. Lemma 2 verifies that $p_c = t^C$ is candidate c's weakly dominant strategy whenever $\rho > \frac{u - \gamma}{1 - \gamma}$. We may therefore restrict to $\rho \leq \frac{u - \gamma}{1 - \gamma}$. This implies

$$\tau_{\text{plu}}^{ac}(g, g, t^C) = \tau_{\text{plu}}^{bc}(g, g, g) = \tau_{ab}^A(g, g) = \tau_{ab}^B(g, g) = 0.$$

In words: if candidates a and b pursue broad campaigning strategies, $p_a = p_b = g$, candidate c loses the election with probability one regardless of her own campaigning strategy, $p_c \in \{g, t^C\}$. We define the following critical thresholds for any $(p_a, p_b) \in \{g, t^A\} \times \{g, t^B\}$:

$$\hat{\tau}_1^{ac}(p_a) \equiv \frac{\gamma u - \alpha u^A(p_a)}{\alpha} + \frac{(\alpha - \gamma)\rho}{\alpha} \tag{40}$$

$$\hat{\tau}_2^{ac}(p_a) \equiv \gamma u - [\alpha u^A(p_a) + (1 - \alpha)u^B(p_a)] + \rho(1 - \gamma) \tag{41}$$

$$\hat{\tau}_1^{bc}(p_b) \equiv \frac{(1 - \alpha)u^B(p_b) - \gamma u}{1 - \alpha} - \frac{\rho(1 - \alpha - \gamma)}{1 - \alpha} \tag{42}$$

$$\hat{\tau}_2^{bc}(p_b) \equiv \alpha u^A(p_b) + (1 - \alpha)u^B(p_b) - \gamma u - \rho(1 - \gamma). \tag{43}$$

Expressions (49) through (52) are the analog of expressions (33) through (36), but under the

supposition that $p_c = g$, rather than $p_c = t^C$. We have:

$$\tau_{\text{plu}}^{ac}(p_a, p_b, g) \equiv \max\{\tau_{ab}^{A}(p_a, p_b), \min\{\hat{\tau}_1^{ac}(p_a), \max\{\hat{\tau}_2^{ac}(p_a), \tau_{ab}^{B}(p_a, p_b)\}\}\}$$

$$\tau_{\text{plu}}^{bc}(p_a, p_b, g) \equiv \min\{\tau_{ab}^{B}(p_a, p_b), \max\{\hat{\tau}_1^{bc}(p_a), \min\{\hat{\tau}_2^{bc}(p_a), \tau_{ab}^{A}(p_a, p_b)\}\}\}.$$

We claim that $p_c = t^C$ is *not* a weakly dominant strategy only if, for any $(p_a, p_b) \neq (g, g)$, $\tau_{\text{plu}}^{bc}(p_a, p_b, g) < \tau_{\text{plu}}^{ac}(p_a, p_b, g)$.

To verify this, notice that if $\tau_{\text{plu}}^{ac}(p_a, p_b, g) \leq \tau_{\text{plu}}^{bc}(p_a, p_b, g)$, then candidate c wins the election with probability zero when the profile is (p_a, p_b, g). Recall that for any (p_a, p_b), $\tau_{\text{plu}}^{ac}(p_a, p_b, g) \geq \tau_{\text{plu}}^{ac}(p_a, p_b, t^A)$ and $\tau_{\text{plu}}^{bc}(p_a, p_b, t^C) \leq \tau_{\text{plu}}^{bc}(p_a, p_b, g)$. This implies that $p_c = t^C$ weakly maximizes c's probability of winning for any (p_a, p_b). Lemma 3 further verified that for any $(p_a, p_b) \neq (g, g)$, $\tau_{\text{plu}}^{ac}(p_a, p_b, t^A) > 0 > \tau_{\text{plu}}^{bc}(p_a, p_b, t^A)$. So, if $\tau_{\text{plu}}^{bc}(p_a, p_b, g) \geq \tau_{\text{plu}}^{ac}(p_a, p_b, g)$ for some $(p_a, p_b) \neq (g, g)$, then c's probability of winning is zero when she chooses $p_c = g$ but strictly positive when she plays $p_c = t^C$, when candidates a and b play p_a and p_B, respectively. This implies that $p_c = t^C$ is c's weakly dominant strategy.

We conclude that if $p_c = t^C$ is *not* c's weakly dominant strategy, then for any triple (p_a, p_b, p_c), candidate a wins if and only if $\tau \geq \tau_{\text{plu}}^{ac}(p_a, p_b, p_c)$, and candidate b wins if and only if $\tau \leq \tau_{\text{plu}}^{bc}(p_a, p_b, p_c)$. Next, recall that $\tau_{\text{plu}}^{bc}(p_a, p_b, t^C) \leq \tau_{\text{plu}}^{bc}(p_a, p_b, g)$ and $\tau_{\text{plu}}^{ac}(p_a, p_b, g) \leq \tau_{\text{plu}}^{ac}(p_a, p_b, t^C)$ for all (p_a, p_b); further, if there exists any pair (p_a, p_b) for which either inequality is strict, then $p_c = t^C$ is c's weakly dominant strategy. So, $p_c = t^C$ is *not* a weakly dominant strategy only if the following is true: for all (p_a, p_b), $\tau_{\text{plu}}^{bc}(p_a, p_b, t^C) = \tau_{\text{plu}}^{bc}(p_a, p_b, g)$ and $\tau_{\text{plu}}^{ac}(p_a, p_b, g) = \tau_{\text{plu}}^{ac}(p_a, p_b, t^C)$. With reference to expressions (33) through (36), recall that

$$\tau_{\text{plu}}^{ac}(p_a, p_b, t^C) \equiv \min\{\tau_1^{ac}(p_a), \max\{\tau_2^{ac}(p_a), \tau_{ab}^{B}(p_a, p_b)\}\}\}$$

$$\tau_{\text{plu}}^{bc}(p_a, p_b, t^C) \equiv \max\{\tau_1^{bc}(p_a), \min\{\tau_2^{bc}(p_a), \tau_{ab}^{A}(p_a, p_b)\}\}\}.$$

Inspection yields that $\tau_1^{ac}(p_a) > \hat{\tau}_1^{ac}(p_a)$, and $\tau_2^{ac}(p_a) > \hat{\tau}_2^{ac}(p_a)$, $\tau_1^{bc}(p_b) < \hat{\tau}_1^{bc}(p_b)$ and $\tau_2^{bc}(p_b) < \hat{\tau}_2^{bc}(p_b)$, this implies that for all (p_a, p_b):

$$\tau_{\text{plu}}^{bc}(p_a, p_b, t^C) = \tau_{\text{plu}}^{bc}(p_a, p_b, g) \iff \tau_{\text{plu}}^{bc}(p_a, p_b, t^C) = \tau_{\text{plu}}^{bc}(p_a, p_b, g) = \tau_{ab}^{A}(p_a, p_b).$$

$$\tau_{\text{plu}}^{ac}(p_a, p_b, t^C) = \tau_{\text{plu}}^{ac}(p_a, p_b, g) \iff \tau_{\text{plu}}^{ac}(p_a, p_b, t^C) = \tau_{\text{plu}}^{ac}(p_a, p_b, g) = \tau_{ab}^{B}(p_a, p_b).$$

Since $\tau_{ab}^{A}(\cdot, t^B) < \tau_{ab}^{A}(\cdot, g)$ and $\tau_{ab}^{B}(g, \cdot) < \tau_{ab}^{B}(t^A, \cdot)$, we conclud that each of the majority candidates a and b has a strictly dominant strategy to pursue a broad campaign, i.e., $p_a = g$ is a's strictly dominant strategy, and $p_b = g$ is b's strictly dominant strategy. Since $\rho \leq \frac{u-\gamma}{1-\gamma}$, for any strategy of candidate c (including any mixed strategy), the strategies of candidates a and b yield an equilibrium outcome that replicates the outcome of the equilibrium characterized in Proposition 1. $\qquad\square$

Claim 2. *If there is a unique pair (p_a, p_b) of pure strategy mutual best responses to $p_c = t^C$, then there does not exist any pair (σ_a, σ_b) with $\sigma_j \in (0, 1)$ for at least one $j \in \{a, b\}$ such that (σ_a, σ_b) are mutual best responses to $p_c = t^C$.*

Proof. Suppose, without loss of generality, that (g, g) is the unique pair of pure strategy mutual best responses to $p_c = t^C$. This implies that the each of $p_a = g$ and $p_b = g$ are strict pure mutual best responses. Suppose there further exists a pair of mutual best responses (σ_g^a, σ_g^b), where $\sigma_g^i = \Pr(p_i = g)$, $i \in \{a, b\}$ and—without loss of generality—$\sigma_g^a \in (0, 1)$. Let $\Pi^i(p_a, p_b)$ denote $i's$ winning probability at an action profile (p_a, p_b, t^C). Since (g, g) are strict mutual best responses:

$$\Pi^a(g, g) > \Pi^a(t^A, g) \tag{44}$$

$$\Pi^b(g, g) > \Pi^b(g, t^B). \tag{45}$$

In the postulated mixed strategy equilibrium with pair (σ_g^a, σ_g^b), a's indifference requires

$$\sigma_g^b[\Pi^a(g, g) - \Pi^a(t^A, g)] = (1 - \sigma_g^b)[\Pi^a(t^A, t^B) - \Pi^a(g, t^B)].$$

Suppose that $\sigma_g^b = 1$. That requires $\Pi^a(g, g) = \Pi^a(t^A, g)$, which contradicts (44). Suppose that $\sigma_g^b \in [0, 1)$. That requires $\Pi^a(t^A, t^B) \geq \Pi^a(g, t^B)$ and also an incentive constraint for candidate b, i.e., that

$$\sigma_g^a[\Pi^b(g, g) - \Pi^b(g, t^B)] \leq (1 - \sigma_g^a)[\Pi^i(t^A, t^B) - \Pi^i(t^A, g)].$$

Combining this incentive constraint with (45) implies $\Pi^i(t^A, t^B) > \Pi^i(t^A, g)$. But this, together

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with $\Pi^a(t^A, t^B) \geq \Pi^a(g, t^B)$, also implies that (t^A, t^B) must be mutual best responses to t^C—a contradiction. $\square$

The proof that (g, t^B) is never a pure strategy equilibrium, combined with Claims 1 and 2 yield Remark 1. $\square$

Proof of Proposition 3. When $\alpha < \alpha^{\text{plu}}$, $p_a = t^A$ and $p_b = t^B$ are strict mutual best responses to t^C, which implies

$$\max\left\{\frac{1-u}{2}, \rho(1-\gamma) + \gamma - u\right\} > \frac{\gamma - (1-\alpha)}{1-\alpha}(1-\rho) = -\tau_1^{bc}(t^B) = -\tau_{\text{plu}}^{bc}(t^A, t^B, t^C).$$

Since $\tau_1^{ac}(t^A) < -\tau_1^{bc}(t^B)$, this further implies

$$\tau_1^{ac}(t^A) < \max\left\{\frac{1-u}{2}, \rho(1-\gamma) + \gamma - u\right\}, \tag{46}$$

Since (g, g) is also a pair of strict mutual best responses for $\rho < \bar{\rho}$, we have:

$$\tau_1^{ac}(t^A) > \min\left\{\frac{u}{2}, \rho(1-\gamma) + \gamma - u\right\}. \tag{47}$$

Suppose $\rho(1-\gamma)+\gamma-u \geq \frac{u}{2}$. Then, (46) and (47) and $u > 1/2$ yield $\frac{u}{2} < \tau_1^{ac}(t^A) < \rho(1-\gamma)+\gamma-u$. But:

$$\gamma F(1) = \alpha F(1 + \tau_1^{ac}(t^A)) < F(u + \tau_1^{ac}(t^A)),$$

which implies $\tau_1^{ac}(t^A) \geq \rho(1-\gamma)+\gamma-u$, since $F(u+\rho(1-\gamma)+\gamma-u) = \gamma F(1)$. This yields a contradiction. We must therefore have $\rho(1-\gamma)+\gamma-u < \frac{u}{2}$. Supposing that $\frac{1-u}{2} \leq \rho(1-\gamma)+\gamma-u < \frac{u}{2}$ also yields a contradiction. We conclude that (46) and (47) hold only if $\rho(1-\gamma)+\gamma-u < \frac{1-u}{2}$, and thus $\rho(1-\gamma)+\gamma-u < \tau_1^{ac}(t^A) < \frac{1-u}{2}$. Thus, $\min\{\frac{1}{2}, \tau_1^{ac}(t^A)\} = \tau_1^{ac}(t^A) = \tau_{\text{plu}}^{ac}(t^A, t^B, t^C)$, and since $\tau_1^{ac}(t^A) > 0$, we have $\tau_{\text{plu}}^{ac}(g, g, t^C) = \max\{0, \gamma - u + \rho(1-\gamma)\} < \tau_1^{ac}(t^A) = \tau_{\text{plu}}^{ac}(t^A, t^B, t^C)$. Since $-\tau_{\text{plu}}^{bc}(t^A, t^B, t^C) = -\tau_1^{bc}(t^B)$, $-\tau_1^{bc}(t^B) > \tau_1^{ac}(t^A)$, and $-\tau_{\text{plu}}^{bc}(g, g, t^C) = \tau_{\text{plu}}^{ac}(g, g, t^C)$, we further conclude that $-\tau_{\text{plu}}^{bc}(t^A, t^B, t^C) > -\tau_{\text{plu}}^{bc}(g, g, t^C)$. $\square$

Proof of Proposition 4. Recognize that for any $p = (p_a, p_b, p_c)$, the candidates' first preferences are given by expressions (28), (29) and (30), as they were under plurality. It is immediate that $p_c = t^C$ is always a (weak) best response to $p_a = p_b = g$. We therefore focus on verifying

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that these strategies of candidates a and b are mutual best responses to c's strategy of $p_c = t^C$. First, we establish some preliminary results that we use repeatedly in this and subsequent propositions. When $p_c = t^C$, candidate c always wins the highest- or the second-highest share of first preferences (Lemma 3). So, when $p_c = t^C$, candidate a wins the election under RCV if and only if

$$\tau \geq \max\{\tau^{ab}(p_a, p_b), \tau^{ac}_{\text{rcv}}(p_a, p_b, t^C)\}$$

where $\tau^{ab}(p_a, p_b)$ is defined in (8) and $\tau^{ac}_{\text{rcv}}(p_a, p_b, t^C)$ is defined in (21). Likewise, candidate b wins if and only if

$$-\tau \geq \max\{-\tau^{ab}(p_a, p_b), -\tau^{bc}_{\text{rcv}}(p_a, p_b, t^C)\}$$

where $\tau^{bc}_{\text{rcv}}(p_a, p_b, t^C)$ is defined in (22).

Observation 1. $\tau^{ab}(p_a, p_b) \in [\tau^A_{ab}(p_a, p_b), \tau^B_{ab}(p_a, p_b, t^C))$.

Proof. That $\tau^{ab}(p_a, p_b) \in [\tau^A_{ab}(p_a, p_b), \tau^B_{ab}(p_a, p_b)]$ follows from the fact that outside of this interval one of the majority candidates receives zero first preference votes. We now show that $\tau^{ab}(p_a, p_b) < \tau^B_{ab}(p_a, p_b)$. Suppose not. Then we must have

$$\alpha F(u^A(p_a) + \tau^B_{ab}(p_a, p_b)) \leq (1 - \alpha)F(u^B(p_b) - \tau^B_{ab}(p_a, p_b)) \tag{48}$$

Since $\tau^B_{ab}(p_a, p_b) = \frac{u^B(p_b) - u^B(p_a)}{2}$, condition (48) becomes

$$\alpha F\left(u^A(p_a) + \frac{u^B(p_b) - u^B(p_a)}{2}\right) \leq (1 - \alpha)F\left(u^B(p_b) - \frac{u^B(p_b) - u^B(p_a)}{2}\right)$$
$$\Leftrightarrow \alpha F\left(u^A(p_a) + \frac{u^B(p_b) - u^B(p_a)}{2}\right) \leq (1 - \alpha)F\left(\frac{u^B(p_b) + u^B(p_a)}{2}\right),$$

which, since $\alpha > 1/2$, requires

$$u^A(p_a) + \frac{u^B(p_b) - u^B(p_a)}{2} < \frac{u^B(p_b) + u^B(p_a)}{2} \Leftrightarrow u^A(p_a) < u^B(p_a),$$

which is impossible since $u^A(p_a) \geq u \geq u^B(p_a)$. $\quad\square$

Corollary 3. $\tau^{ab}(p_a, p_b, t^C) = \max\{\tau^A_{ab}(p_a, p_b), \tau^{ab}_{int}(p_a, p_b)\}$, *where*

$$\tau^{ab}_{int}(p_a, p_b) \equiv (1 - \alpha)u^B(p_b) - \alpha u^A(p_a) + (2\alpha - 1)\rho,$$

solves $\alpha F(u^A(p_a) + \tau) = (1 - \alpha)F(u^B(p_b) - \tau)$.

When candidate c targets her base, the following observation verifies that unless both majority candidates also target their bases, the following holds: if group-A voters prefer a over b, then group-B voters prefer a over c, and if group-B voters prefer b over a, then group-A voters prefer b over c.

Observation 2. *For any* $(p_a, p_b) \neq (t^A, t^B)$: $\tau^B_{ac}(p_a, t^C) < \tau^A_{ab}(p_a, p_b)$ *and* $\tau^B_{ab}(p_a, p_b) < \tau^A_{bc}(p_b, t^C)$.

Proof. We prove the first claim, leaving the second to the reader. We have $\tau^B_{ac}(p_a, t^C) < \tau^A_{ab}(p_a, p_b)$ if and only if $0 - u^B(p_a) - \theta < \frac{u^A(p_b) - u^A(p_a)}{2}$. If $p_a = g$, then this condition holds if $-u - \theta < \frac{0-u}{2}$, which is true for all $\theta > 0$; if $p_b = g$, then the condition holds if $-\theta < \frac{u-1}{2}$, which holds because $\theta > u/2$ and $u > 1/2$. $\square$

Observation 3. *If* $\theta > 1/2$, *Observation* 2 *strengthens: for any* (p_a, p_b), $\tau^B_{ac}(p_a, t^C) < \tau^A_{ab}(p_a, p_b)$ *and* $\tau^B_{ab}(p_a, p_b) < \tau^A_{bc}(p_b, t^C)$.

Proof. We need only show that $\tau^B_{ac}(t^A, t^C) < \tau^A_{ab}(t^A, t^B)$ and $\tau^B_{ab}(t^A, t^B) < \tau^A_{bc}(t^B, t^C)$, and we prove the first claim since the second follows the same reasoning. We have $\tau^B_{ac}(p_a, t^C) < \tau^A_{ab}(p_a, p_b)$ if and only if $-\theta < \frac{0-1}{2}$, i.e., if and only if $\theta > \frac{1}{2}$. $\square$

The next observation verifies that when candidate c targets her core supporters with policy $p_c = t^C$, the critical threshold $\tau^{ac}_{rcv}(p_a, p_b, t^C)$ above which a defeats c does not depend on b's policy; similarly, the critical threshold $\tau^{bc}_{rcv}(p_a, p_b, t^C)$ below which b defeats c does not depend on a's policy.

Observation 4. *For* $p_a \in \{g, t^A\}$, $\tau^{ac}_{rcv}(p_a, g, t^C) = \tau^{ac}_{rcv}(p_a, t^B, t^C)$. *For* $p_b \in \{g, t^B\}$, $\tau^{bc}_{rcv}(g, p_b, t^C) = \tau^{bc}_{rcv}(t^A, p_b, t^C)$.

Proof. We prove the result for a's thresholds, leaving b's to the reader. We start by verifying that $\tau_{\text{rcv}}^{ac}(t^A, g, t^C) = \tau_{\text{rcv}}^{ac}(t^A, t^B, t^C)$. Notice that, for any $p_b \in \{g, t^B\}$:

$$\tau_{\text{rcv}}^{ac}(t^A, p_b, t^C) = \max\left\{\tau_{ab}^A(t^A, p_b), \tau_{ac}^B(t^A, t^C), \min\{\tau_1^{ac}(t^A), \tau_2^{ac}(t^A)\}\right\}.$$

To see why, recognize first that if $\tau < \tau_{ab}^A(p_a, p_b)$, a wins the fewest first preferences (in particular, zero first preferences) and therefore loses the election. Recognizing that $\tau_{ac}^B(t^A, t^C) = -\theta$, if $\tau \leq \tau_{ac}^B(t^A, t^C)$, then a's total first and second preferences are $\alpha F(1+\tau) < \alpha F(1+0) < \gamma F(1) + (1-\alpha)F(0-\theta)$, and so a loses the election. Since $\tau \geq \tau_{ac}^B(t^A, t^C)$ implies that a's sum of first and second preferences is $\alpha F(1+\tau) + (1-\alpha)F(0+\tau)$, the requirement $\tau \geq \min\{\tau_1^{ac}(t^A), \tau_2^{ac}(t^A)\}$ is immediate. Under our assumption $\rho > \rho_{\min}$, we have $\tau_{ab}^A(t^A, p_b) < \tau_2^{ac}(t^A)$ for any (t^A, p_b), and since $\tau_1^{ac}(t^A) > 0 > \tau_{ab}^A(t^A, p_b)$ for either $p_b \in \{g, t^B\}$, we conclude that $\min\{\tau_1^{ac}(t^A), \tau_2^{ac}(t^A)\} > \tau_{ab}^A(t^A, p_b)$. Further, since $\tau_{ac}^B(t^A, t^C) = -\theta \leq -\frac{u}{2} < \frac{u-1}{2} < \min\{\tau_2^{ac}(p_a), \tau_1^{ac}(p_a)\}$, we conclude that $\tau_{\text{rcv}}^{ac}(t^A, p_b, t^C) = \min\{\tau_1^{ac}(t^A), \tau_2^{ac}(t^A)\}$. This verifies that $\tau_{\text{rcv}}^{ac}(t^A, g, t^C) = \tau_{\text{rcv}}^{ac}(t^A, t^B, t^C)$.

We next verify that $\tau_{\text{rcv}}^{ac}(g, g, t^C) = \tau_{\text{rcv}}^{ac}(g, t^B, t^C)$. Notice that $\tau_{ac}^A(g, t^C) = \tau_{ac}^B(g, t^C) = -u - \theta < \frac{1}{2}$ and $\tau_{bc}^A(g, t^C) = \tau_{bc}^B(g, t^C) = u + \theta > \frac{1}{2}$. This implies that if $p_a = g$ and $p_c = t^C$, group-B voters always prefer a to c, and similarly if $p_b = g$ and $p_c = t^C$, group A voters always prefer b to c. This implies that

$$v_a^f(g, p_b, t^C, \tau) = \alpha\phi \max\{u + \tau - \rho, 0\}\mathbb{I}\left\{\tau \geq \frac{u^A(p_b) - u}{2}\right\}$$
$$+ (1-\alpha)\phi \max\{u + \tau - \rho, 0\}\left\{\tau > \frac{u^B(p_b) - u}{2}\right\},$$

and

$$v_a^s(g, p_b, t^C, \tau) = \alpha\phi \max\{u + \tau - \rho, 0\}\mathbb{I}\left\{\tau < \frac{u^A(p_b) - u}{2}\right\}$$
$$+ (1-\alpha)\phi \max\{u + \tau - \rho, 0\}\left\{\tau \leq \frac{u^B(p_b) - u}{2}\right\},$$

implying that

$$v_a^f(g, p_b, t^C, \tau) + v_a^s(g, p_b, t^C, \tau) = \phi \max\{u + \tau - \rho, 0\},$$

which is independent of b's policy p_b. This implies that $\tau_{\text{rcv}}^{ac}(g, g, t^C) = \tau_{\text{rcv}}^{ac}(g, t^B, t^C)$, as was to

be shown. □

Corollary 4. *We have*

$$\max\{\tau^{ab}(g,g), \tau^{ac}_{rcv}(g, \cdot, t^C)\} = \max\{0, \gamma - u + \rho(1-\gamma)\} = \max\{-\tau^{ab}(g,g), -\tau^{bc}_{rcv}(\cdot, g, t^C)\}.$$

Observation 5. *For any* (p_a, p_b), $\tau^B_{ac}(p_a, t^C) < 0$ *and* $\tau^B_{ac}(p_a, t^C) > 0$.

Proof. Follows from $\theta > 0$. □

Notice that $\rho > \rho_{\min}$ implies $\gamma - \alpha + \rho(1-\gamma) > (u-1)/2 > -\theta$, and that $1 - \alpha - \gamma - \rho(1-\gamma) < +\theta$. Together with the previous observations, the following is easily verified:

$$\max\{\tau^{ab}(g, t^B), \tau^{ac}_{rcv}(g, \cdot, t^C)\} = \max\left\{-\frac{u}{2}, 1 - \alpha - \alpha u + (2\alpha - 1)\rho, \gamma - u + \rho(1-\gamma)\right\}$$

$$\max\{-\tau^{ab}(t^A, g), -\tau^{bc}_{rcv}(\cdot, g, t^C)\} = \max\left\{\min\left\{\frac{1-u}{2}, \alpha - (1-\alpha)u - (2\alpha - 1)\rho\right\}, \gamma - u + \rho(1-\gamma)\right\}.$$

$$\tau^{ac}_{rcv}(t^A, \cdot, t^C) = \min\left\{\left(\frac{\gamma}{\alpha} - 1\right)(1 - \rho), \gamma(1 - \rho) - \alpha + \rho\right\}$$

$$= \begin{cases} \left(\frac{\gamma}{\alpha} - 1\right)(1 - \rho) & \text{if } \gamma(1 - \rho) \leq \alpha \\ \gamma(1 - \rho) - \alpha + \rho & \text{if } \gamma(1 - \rho) > \alpha \end{cases}$$

$$-\tau^{bc}_{rcv}(\cdot, t^B, t^C) = \min\left\{\left(\frac{\gamma}{1-\alpha} - 1\right)(1 - \rho), \gamma(1 - \rho) - 1 + \alpha + \rho\right\}$$

$$= \begin{cases} \left(\frac{\gamma}{1-\alpha} - 1\right)(1 - \rho) & \text{if } \gamma(1 - \rho) \leq 1 - \alpha \\ \gamma(1 - \rho) - 1 + \alpha + \rho & \text{if } \gamma(1 - \rho) > 1 - \alpha. \end{cases}$$

Observation 6. *We have* $\tau^{ac}_{rcv}(t^A, \cdot, t^C) < -\tau^{bc}_{rcv}(\cdot, t^B, t^C)$.

Proof. When $\gamma(1 - \rho) \notin (1 - \alpha, \alpha)$, the claim follows by inspection and $\alpha > 1/2$. When instead $\gamma(1 - \rho) \in (1 - \alpha, \alpha)$, we have that:

$$-\tau^{bc}_{rcv}(\cdot, t^B, t^C) = \gamma(1 - \rho) - 1 + \alpha + \rho > 1 - (1 - \rho) > \frac{\gamma}{\alpha}(1 - \rho) - (1 - \rho) = \tau^{ac}_{rcv}(t^A, \cdot, t^C).$$

This completes the proof. □

We now establish the proposition. Given that c chooses $p_c = t^C$, $p_a = g$ and $p_b = g$ are mutual best responses if and only if:

$$
\begin{cases}
\max\{\tau^{ab}(g,g), \tau^{ac}_{\text{rcv}}(g,g,t^C)\} \leq \max\{\tau^{ab}(t^A,g), \tau^{ac}_{\text{rcv}}(t^A,g,t^C)\} \\
\max\{-\tau^{ab}(g,g), -\tau^{bc}_{\text{rcv}}(g,g,t^C)\} \leq \max\{-\tau^{ab}(g,t^B), -\tau^{bc}_{\text{rcv}}(g,t^B,t^C)\}
\end{cases}
$$

$$
\Leftrightarrow \max\{0, \gamma - u + \rho(1-\gamma)\} \leq \min
\left\{
\begin{array}{c}
\max\{\max\{(1-\alpha)u - \alpha + (2\alpha-1)\rho, \frac{u-1}{2}\}, \tau^{ac}_{\text{rcv}}(t^A,g,t^C)\}, \\
\max\{-\max\{1-\alpha - \alpha u + (2\alpha-1)\rho, -\frac{u}{2}\}, -\tau^{bc}_{\text{rcv}}(g,t^B,t^C)\}
\end{array}
\right\}.
$$

Since $(1-\alpha)u - \alpha + (2\alpha-1)\rho \leq (1-\alpha)u - \alpha + (2\alpha-1)\frac{u}{2} < 0$ and $-\tau^{bc}_{\text{rcv}}(g,t^B,t^C) > \tau^{ac}_{\text{rcv}}(t^A,g,t^C)$, the condition further simplifies to:

$$
\max\{0, \gamma - u + \rho(1-\gamma)\} \leq \max\left\{\frac{u-1}{2}, \tau^{ac}_{\text{rcv}}(t^A,g,t^C)\right\}
$$

$$
= \max\left\{\frac{u-1}{2}, \min\left\{\frac{\gamma-\alpha}{\alpha}(1-\rho), \gamma - \alpha + \rho(1-\gamma)\right\}\right\}.
$$

If $\gamma - u + \rho(1-\gamma) \geq 0$, then $\gamma - \alpha + \rho(1-\gamma) > 0$, so the condition is equivalent to $\gamma - u + \rho(1-\gamma) \leq \frac{\gamma-\alpha}{\alpha}(1-\rho)$, i.e., $\rho \leq \bar{\rho}$. If $\gamma - u + \rho(1-\gamma) < 0$, then the condition is equivalent to $\gamma - \alpha + \rho(1-\gamma) \geq 0$, i.e., $\rho \geq \underline{\rho}$. $\square$

The next lemma verifies that if $\theta \geq (u+1)/2$, $p_c = t^C$ is always best response.

Lemma 4. *Let $\theta \geq (u+1)/2$. For all other primitives, $p_c = t^C$ is a weakly dominant strategy.*

Proof. To start, recognize that for any $(p_a, p_b) \in \{g, t^A\} \times \{g, t^B\}$, any $p_c \in \{g, t^C\}$ and $\theta \geq (u+1)/2$, we have

$$
\tau^B_{ac}(p_a, p_c) \leq u - \theta \leq u - \frac{u+1}{2} = \frac{u-1}{2} \leq \tau^A_{ab}(p_a, p_b).
$$

In words: for any policy platforms, whenever a wins strictly positive first preferences, she is also preferred to candidate c by group-B voters. Similarly, for any $(p_a, p_b) \in \{g, t^A\} \times \{g, t^B\}$, any $p_c \in \{g, t^C\}$ and $\theta \geq (u+1)/2$:

$$
\tau^A_{bc}(p_b, p_c) \geq \theta - u \geq \frac{u+1}{2} - u = \frac{1-u}{2} \geq \tau^B_{ab}(p_a, p_b).
$$

Let:

$$\tau_1^{ac}(p_a, p_c) \equiv \frac{\gamma u^C(p_c) - \alpha u^A(p_a)}{\alpha} + \frac{(\alpha - \gamma)\rho}{\alpha} \tag{49}$$

$$\tau_2^{ac}(p_a, p_c) \equiv \gamma u^C(p_c) - [\alpha u^A(p_a) + (1 - \alpha)u^B(p_a)] + \rho(1 - \gamma) \tag{50}$$

$$\tau_1^{bc}(p_b, p_c) \equiv \frac{(1 - \alpha)u^B(p_b) - \gamma u^C(p_c)}{1 - \alpha} - \frac{\rho(1 - \alpha - \gamma)}{1 - \alpha} \tag{51}$$

$$\tau_2^{bc}(p_b, p_c) \equiv \alpha u^A(p_b) + (1 - \alpha)u^B(p_b) - \gamma u^C(p_c) - \rho(1 - \gamma). \tag{52}$$

For any (p_a, p_b), c wins the election with policy $p_c = g$ *only if*

$$\min\{\tau^{ab}(p_a, p_b), \max\{\tau_1^{bc}(p_b, g), \tau_2^{bc}(p_b, g)\}\}$$
$$\leq \tau \leq \max\{\tau^{ab}(p_a, p_b), \tau_{ac}^B(p_a, g), \min\{\tau_1^a(p_a, g), \tau_2^{ac}(p_a, g)\}\}. \tag{53}$$

To understand why, we begin with the first inequality. For any $(p_a, p_b) \in \{g, t^A\} \times \{g, t^B\}$ and $\theta > \frac{u+1}{2}$, $\tau^{ab}(p_a, p_b) \leq (1-u)/2 < \theta - u \leq \tau_{bc}^A(p_b, g)$. So, if candidate b wins more first preferences than candidate a, candidate b also wins any second preferences cast by group-A voters, and even their first preferences if $\tau < \tau_{ab}^A(p_a, p_b)$. If, in addition, $\alpha F(0-\tau) + (1-\alpha)F(1-\tau) \geq \gamma F(u)$, which is equivalent to $\tau < \max\{\tau_1^{bc}(t^B, g), \tau_2^{bc}(t^B, g)\}$, candidate b's combined first and second preferences exceed candidate c's. The second inequality follows a similar logic.

Next, recognize that for $\theta \geq (u+1)/2$ when c locates at $p_c = t^C$, instead, she wins whenever:

$$\min\{\tau^{ab}(p_a, p_b), \max\{\tau_1^{bc}(p_b, t^C), \tau_2^{bc}(p_b, t^C)\}\}$$
$$\leq \tau \leq \max\{\tau^{ab}(p_a, p_b), \min\{\tau_1^a(p_a, t^C), \tau_2^{ac}(p_a, t^C)\}\}. \tag{54}$$

Since $\min\{\tau_1^a(t^A, t^C), \tau_2^{ac}(t^A, t^C)\} > \min\{\tau_1^a(t^A, g), \tau_2^{ac}(t^A, g)\}$ and $\theta > (u+1)/2$ implies that $\tau_{ac}^B(p_a, g) < u - \theta < (u-1)/2 < \tau_2^{ac}(p_a, t^C)$, we conclude that the set of shocks defined in (53) is a subset of the corresponding set of shocks defined in (54) for all (p_a, p_b), and moreover a *strict* subset when $p_a = t^A$ and $p_b = g$, since $\tau^{ab}(t^A, g) = \frac{u-1}{2} < \min\{\tau_1^{ac}(t^A, g), \tau_2^{ac}(t^A, g)\}$ under our assumption that $\rho > \rho_{\min}$. □

Proof of Proposition 5. We proceed by a sequence of lemmas.

Lemma 5. *If $\theta \geq \frac{1+u}{2}$, there is never an equilibrium in which $p_a = g$, $p_b = t^B$ and $p_c = t^C$.*

Proof. Since $\theta \geq \frac{1}{2}$, Observation 3 applies. Thus, $p_a = g$ and $p_b = t^B$ are mutual best responses to $p_c = t^C$ if and only if

$$
\begin{cases}
\max\{\tau^{ab}(g, t^B), \tau^{ac}_{\mathrm{rcv}}(g, t^B, t^C)\} \leq \max\{\tau^{ab}(t^A, t^B), \tau^{ac}_{\mathrm{rcv}}(t^A, t^B, t^C)\} \\
\max\{-\tau^{ab}(g, t^B), -\tau^{bc}_{\mathrm{rcv}}(g, t^B, t^C)\} \leq \max\{-\tau^{ab}(g, g), -\tau^{bc}_{\mathrm{rcv}}(g, g, t^C)\}
\end{cases}
$$

$$
\Leftrightarrow
\begin{cases}
\max\left\{1 - \alpha - \alpha u + (2\alpha - 1)\rho, -\frac{u}{2}, \tau^{ac}_{\mathrm{rcv}}(g, t^B, t^C)\right\} \leq \max\{-(2\alpha - 1)(1 - \rho), -\frac{1}{2}, \tau^{ac}_{\mathrm{rcv}}(t^A, t^B, t^C)\} \\
\max\{-\max\{1 - \alpha - \alpha u + (2\alpha - 1)\rho, -\frac{u}{2}\}, -\tau^{bc}_{\mathrm{rcv}}(g, t^B, t^C)\} \leq \max\left\{0, \tau^{ac}_{\mathrm{rcv}}(g, g, t^C)\right\}.
\end{cases}
$$

Since $-\frac{u}{2} > -\frac{1}{2}$ and $1 - \alpha - \alpha u + (2\alpha - 1)\rho > -(2\alpha - 1)(1 - \rho)$, the first condition (i.e., candidate a's constraint) requires

$$
\tau^{ac}_{\mathrm{rcv}}(g, t^B, t^C) \leq \tau^{ac}_{\mathrm{rcv}}(t^A, t^B, t^C),
$$

There are two possible cases. If $1 - \alpha - \alpha u + (2\alpha - 1)\rho \leq 0$, the second condition (i.e., candidate b's constraint) requires

$$
\tau^{ac}_{\mathrm{rcv}}(g, g, t^C) \geq -\tau^{bc}_{\mathrm{rcv}}(g, t^B, t^C),
$$

and since $\tau^{ac}_{\mathrm{rcv}}(g, g, t^C) = \tau^{ac}_{\mathrm{rcv}}(g, t^B, t^C)$ and $\tau^{ac}_{\mathrm{rcv}}(t^A, g, t^C) = \tau^{ac}_{\mathrm{rcv}}(t^A, t^B, t^C)$, we therefore require $\tau^{ac}_{\mathrm{rcv}}(t^A, g, t^C) \geq -\tau^{bc}_{\mathrm{rcv}}(g, t^B, t^C)$, which is always false. If, instead, $1 - \alpha - \alpha u + (2\alpha - 1)\rho > 0$, then the first condition implies

$$
\tau^{ac}_{\mathrm{rcv}}(t^A, t^B, t^C) \geq \max\left\{1 - \alpha - \alpha u + (2\alpha - 1)\rho, \tau^{ac}_{\mathrm{rcv}}(g, t^B, t^C)\right\} \geq \max\left\{0, \tau^{ac}_{\mathrm{rcv}}(g, t^B, t^C)\right\} \quad (55)
$$

and the second condition requires $\max\left\{0, \tau^{ac}_{\mathrm{rcv}}(g, g, t^C)\right\} \geq -\tau^{bc}_{\mathrm{rcv}}(g, t^B, t^C)$. Since $\tau^{ac}_{\mathrm{rcv}}(g, g, t^C) = \tau^{ac}_{\mathrm{rcv}}(g, t^B, t^C)$, the second condition therefore requires $\max\left\{0, \tau^{ac}_{\mathrm{rcv}}(g, t^B, t^C)\right\} \geq -\tau^{bc}_{\mathrm{rcv}}(g, t^B, t^C)$. Combining this with (55) yields $\tau^{ac}_{\mathrm{rcv}}(t^A, t^B, t^C) \geq -\tau^{bc}_{\mathrm{rcv}}(g, t^B, t^C)$, which is always false. $\qquad \square$

We next define the following thresholds:

$$
\alpha^{\mathrm{rcv}}_1 \equiv \frac{1}{2} + \frac{1 - u}{4(1 - \rho)}
$$

$$\alpha_2^{\text{rcv}} \equiv \max\left\{ \frac{1-u}{2} + (1-\rho)(1-\gamma), 1 - \frac{\gamma(1-\rho)}{1-\rho+\max\left\{\frac{1-u}{2}, \gamma(1-\rho)+\rho-u\right\}}, \right\}$$

Lemma 6. *If $\theta \geq (u+1)/2$, then $(p_a, p_b, p_C) = (t^A, g, t^C)$ is an equilibrium if and only if either*

(i) $\rho \leq \underline{\rho}$ and $\alpha \geq \alpha_1^{\text{rcv}}$, or

(ii) $\rho \geq \overline{\rho}$ and $\alpha \geq \alpha_2^{\text{rcv}}$.

Proof. Lemma 4 verifies that candidate c's strategy is a best response, so we focus on the strategies of candidates a and b. Recognize that $p_a = t^A$ and $p_b = g$ are mutual best responses if and only if:

$$\begin{cases} \max\{\tau^{ab}(t^A, g), \tau_{\text{rcv}}^{ac}(t^A, g, t^C)\} \leq \max\{\tau^{ab}(g, g), \tau_{\text{rcv}}^{ac}(g, g, t^C)\} \\ \max\{-\tau^{ab}(t^A, g,), -\tau_{\text{rcv}}^{bc}(t^A, g, t^C)\} \leq \max\{-\tau^{ab}(t^A, t^B), -\tau_{\text{rcv}}^{bc}(t^A, t^B, t^C).\} \end{cases}$$

Our analysis from the previous lemma yields that the first condition is equivalent to $\rho \notin (\underline{\rho}, \overline{\rho})$. Since

$$-\tau^{ab}(t^A, g) = \min\left\{ \frac{1-u}{2}, \alpha - (1-\alpha)u - (2\alpha - 1)\rho \right\}$$

and

$$\alpha - (1-\alpha)u - (2\alpha - 1)\rho > \alpha - (1-\alpha)u - (2\alpha - 1)\frac{u}{2} = \alpha - \frac{u}{2} > \frac{1-u}{2},$$

the second condition is equivalent to

$$\max\left\{ \frac{1-u}{2}, (1-\rho)\gamma + \rho - u \right\} \leq \max\left\{ \begin{array}{c} \min\left\{\frac{1}{2}, (2\alpha - 1)(1-\rho)\right\}, \\ \min\left\{\left(\frac{\gamma}{1-\alpha} - 1\right)(1-\rho), \gamma(1-\rho) - 1 + \alpha + \rho\right\} \end{array} \right\}.$$

We consider the two restrictions on ρ separately.

Case 1: $\rho \leq \underline{\rho}$. In this case, we have $(1-\rho)\gamma + \rho - u \leq \alpha - u < \frac{1-u}{2}$ and $(1-\rho)\gamma \geq \frac{\gamma}{1-\gamma}(1-\alpha) > 1 - \alpha$, which implies

$$\left(\frac{\gamma}{1-\alpha} - 1 \right)(1-\rho) > \gamma(1-\rho) - 1 + \alpha + \rho.$$

Using again $(1-\rho)\gamma + \rho - u \leq \alpha$ we obtain $\gamma(1-\rho) - 1 + \alpha + \rho \leq 2\alpha - 1$. Since, in addition, $\underline{\rho} < 0$, we also have $2\alpha - 1 < (2\alpha - 1)(1-\rho)$. Putting everything together, the condition

simplifies to $\frac{1-u}{2} \leq (2\alpha - 1)(1 - \rho)$, or $\alpha \geq \alpha_1^{\text{rcv}}$.

Case 2: $\rho \geq \bar{\rho}$. This case requires, equivalently, that $\gamma(1 - \rho) \leq \alpha\frac{1-u}{1-\alpha}$ and $\gamma(1-\rho) + \rho \geq 1 - \alpha\frac{1-u}{1-\alpha}\frac{1-\gamma}{\gamma}$. Notice that in this case we have

$$\gamma(1-\rho) - 1 + \alpha + \rho = (1-\rho)\left(\gamma - 1 + \frac{\alpha}{1-\rho}\right) \geq (1-\rho)\left(\gamma - 1 + \gamma\frac{1-\alpha}{1-u}\right) > (2\alpha - 1)(1 - \rho).$$

Since in addition $\left(\frac{\gamma}{1-\alpha} - 1\right)(1-\rho) > (2\alpha - 1)(1 - \rho)$, we obtain

$$\min\left\{\left(\frac{\gamma}{1-\alpha} - 1\right)(1-\rho), \gamma(1-\rho) - 1 + \alpha + \rho\right\} > (2\alpha - 1)(1 - \rho)$$

and the condition simplifies to

$$\max\left\{\frac{1-u}{2}, (1-\rho)\gamma + \rho - u\right\} \leq \min\left\{\left(\frac{\gamma}{1-\alpha} - 1\right)(1-\rho), \gamma(1-\rho) - 1 + \alpha + \rho\right\}$$

Since $(1-\rho)\gamma + \rho - u < (1-\rho)\gamma + \rho - 1 + \alpha$, it is equivalent to

$$\frac{1-u}{2} \leq \gamma(1-\rho) - 1 + \alpha + \rho \quad \text{and} \quad \max\left\{\frac{1-u}{2}, (1-\rho)\gamma + \rho - u\right\} \leq \left(\frac{\gamma}{1-\alpha} - 1\right)(1-\rho),$$

which is equivalent to $\alpha \geq \alpha_2^{\text{rcv}}$. $\qquad\square$

Next, define the following thresholds:

$$\alpha_3^{\text{rcv}} = \max\left\{\frac{1-u}{2} + (1-\gamma)(1-\rho), 1 - \frac{\gamma(1-\rho)}{\frac{1-u}{2} + (1-\rho)}\right\}$$

$$\alpha_4^{\text{rcv}} = \frac{1}{2} + \min\left\{\frac{1-u}{4(1-\rho)}, \left[(1-\rho)(1-\gamma) - \frac{u}{2}\right]\min\left\{1, \frac{1}{u - 2\rho}\right\}\right\}$$

$$\alpha_5^{\text{rcv}} = \alpha_4^{\text{rcv}}\mathbb{I}\left\{\rho \leq 1 - \frac{\alpha}{\gamma}\right\} + \alpha_3^{\text{rcv}}\mathbb{I}\left\{\rho > 1 - \frac{\alpha}{\gamma}\right\}$$

Lemma 7. *If $\theta \geq \frac{u+1}{2}$, then $(p_a, p_b, p_c) = (t^A, t^B, t^C)$ is an equilibrium if and only if either*

(i) $\rho \leq \underline{\rho}$ and $\alpha \leq \alpha_1^{rcv}$, or

(ii) $\rho \in (\underline{\rho}, \bar{\rho}]$ and $\alpha \leq \alpha_5^{rcv}$, or

(iii) $\rho \geq \bar{\rho}$ and $\alpha \leq \alpha_2^{rcv}$.

Proof. Lemma 4 verifies that candidate c's strategy is a best response, so we focus on verifying that $p_A = t^A$ and $p_B = t^B$ are mutual best responses. This is true if and only if

$$\begin{cases} \max\{\tau^{ab}(t^A, t^B), \tau^{ac}_{\text{rcv}}(t^A, t^B, t^C)\} \leq \max\{\tau^{ab}(g, t^B), \tau^{ac}_{\text{rcv}}(g, t^B, t^C)\} \\ \max\{-\tau^{ab}(t^A, t^B), -\tau^{bc}_{\text{rcv}}(t^A, t^B, t^C)\} \leq \max\{-\tau^{ab}(t^A, g), -\tau^{bc}_{\text{rcv}}(t^A, g, t^C)\} \end{cases}$$

$$\Leftrightarrow \begin{cases} \max\left\{-\frac{1}{2}, -(2\alpha - 1)(1 - \rho), \tau^{ac}_{\text{rcv}}(t^A, t^B, t^C)\right\} \leq \max\left\{-\frac{u}{2}, 1 - \alpha - \alpha u + (2\alpha - 1)\rho, \tau^{ac}_{\text{rcv}}(g, t^B, t^C)\right\} \\ \max\{\min\left\{\frac{1}{2}, (2\alpha - 1)(1 - \rho)\right\}, -\tau^{bc}_{\text{rcv}}(t^A, t^B, t^C)\} \leq \max\{-\tau^{ab}(t^A, g, t^C), -\tau^{bc}_{\text{rcv}}(t^A, g, t^C)\}. \end{cases}$$

Using the fact that $-\tau^{ab}(t^A, g) = \frac{1-u}{2}$, and that $\tau^{ac}_{\text{rcv}}(g, \cdot, t^C) = -\tau^{bc}_{\text{rcv}}(\cdot, g, t^C)$, and further recognizing that $1 - \alpha - \alpha u + (2\alpha - 1)\rho > -(2\alpha - 1)(1 - \rho)$, the conditions become:

$$\begin{cases} \tau^{ac}_{\text{rcv}}(t^A, t^B, t^C) \leq \max\left\{-\frac{u}{2}, 1 - \alpha - \alpha u + (2\alpha - 1)\rho, \tau^{ac}_{\text{rcv}}(g, t^B, t^C)\right\} \\ \max\{\min\left\{\frac{1}{2}, (2\alpha - 1)(1 - \rho)\right\}, -\tau^{bc}_{\text{rcv}}(t^A, t^B, t^C)\} \leq \max\left\{\frac{1-u}{2}, \tau^{ac}_{\text{rcv}}(g, t^B, t^C)\right\} \end{cases}$$

Since $\tau^{ac}_{\text{rcv}}(g, \cdot, t^C) < 1 - u < 1/2$, the conditions further simplify to

$$\begin{cases} \tau^{ac}_{\text{rcv}}(t^A, t^B, t^C) \leq \max\left\{-\frac{u}{2}, 1 - \alpha - \alpha u + (2\alpha - 1)\rho, \tau^{ac}_{\text{rcv}}(g, t^B, t^C)\right\}. \\ \max\{(2\alpha - 1)(1 - \rho), -\tau^{bc}_{\text{rcv}}(t^A, t^B, t^C)\} \leq \max\left\{\frac{1-u}{2}, \tau^{ac}_{\text{rcv}}(g, t^B, t^C)\right\}. \end{cases} \tag{56}$$

We consider three cases separately.

Case 1: $\rho \leq \underline{\rho}$. In this case we have (i) $\rho < 0$ and (ii)

$$-\tau^{bc}_{\text{rcv}}(\cdot, t^B, t^C) > \tau^{ac}_{\text{rcv}}(t^A, \cdot, t^C) = \gamma(1 - \rho) + \rho - \alpha > \gamma(1 - \rho) + \rho - u = \tau^{ac}_{\text{rcv}}(g, \cdot, t^C)$$

which implies that we can re-write the equilibrium conditions as follows:

$$\begin{cases} \tau^{ac}_{\text{rcv}}(t^A, t^B, t^C) \leq \max\left\{-\frac{u}{2}, 1 - \alpha - \alpha u + (2\alpha - 1)\rho\right\} \\ \max\{(2\alpha - 1)(1 - \rho), -\tau^{bc}_{\text{rcv}}(t^A, t^B, t^C)\} \leq \frac{1-u}{2} \end{cases}$$

Observing that $(2\alpha - 1)(1 - \rho) \le \frac{1-u}{2}$ implies

$$1 - \alpha - \alpha u + (2\alpha - 1)\rho \ge 1 - \alpha - \alpha u + 2\alpha - 1 - \frac{1-u}{2} = \left(\alpha - \frac{1}{2}\right)(1 - u) > 0$$

we cab re-write the equilibrium conditions:

$$\begin{cases} \alpha \le \gamma(1 - \rho) \\ \tau_{\text{rcv}}^{ac}(t^A, t^B, t^C) \le 1 - \alpha - \alpha u + (2\alpha - 1)\rho \\ (2\alpha - 1)(1 - \rho) \le \frac{1-u}{2} \\ -\tau_{\text{rcv}}^{bc}(t^A, t^B, t^C) \le \frac{1-u}{2} \end{cases} \Leftrightarrow \begin{cases} \alpha \le \gamma(1 - \rho) \\ \alpha \le \frac{1}{2} + \frac{(1-\gamma)(1-\rho) - \frac{u}{2}}{u - 2\rho} \\ \alpha \le \frac{1}{2} + \frac{1-u}{4(1-\rho)} = \alpha_1^{\text{rcv}} \\ \alpha \le \frac{1}{2} + (1 - \gamma)(1 - \rho) - \frac{u}{2}. \end{cases} \tag{57}$$

We now show that when $\rho \le \underline{\rho}$ and (57) is feasible, then $\alpha \le \alpha_1^{\text{rcv}} \Rightarrow$ (57). First $\alpha \le \gamma(1 - \rho)$ is equivalent to $\rho \le 1 - \frac{\alpha}{\gamma}$, which is implied by $\rho \le \underline{\rho}$. Second, $\rho \le \underline{\rho} \Leftrightarrow \alpha \ge \rho + \gamma(1 - \rho)$. Together with $\alpha \le \alpha_1^{\text{rcv}}$, this implies $(1 - \gamma)(1 - \rho) \ge 1 - \alpha_1^{\text{rcv}}$. Hence, we obtain

$$\frac{1}{2} + (1 - \gamma)(1 - \rho) - \frac{u}{2} - \alpha_1^{\text{rcv}} \ge \frac{1-u}{2} + 1 - 2\alpha_1^{\text{rcv}} = \frac{1-u}{2}\left(1 - \frac{1}{1-\rho}\right) > 0$$

where the last inequality follows from $\underline{\rho} < 0$. Third, using again $(1 - \gamma)(1 - \rho) \ge 1 - \alpha_1^{\text{rcv}}$, we obtain that

$$\frac{1}{2} + \frac{(1-\gamma)(1-\rho) - \frac{u}{2}}{u - 2\rho} - \alpha_1^{\text{rcv}} \ge \frac{1}{2} + \frac{\frac{1}{2} + \frac{1}{2} - \alpha_1^{\text{rcv}} - \frac{u}{2}}{u - 2\rho} - \alpha_1^{\text{rcv}}$$
$$= \left(\frac{1}{2} - \alpha_1^{\text{rcv}}\right)\left(1 + \frac{1}{u - 2\rho}\right) + \frac{1-u}{2}\frac{1}{u - 2\rho}$$
$$\propto -\frac{1-u}{4(1-\rho)}(1 + u - 2\rho) + \frac{1-u}{2} \propto 1 - \frac{1 + u - 2\rho}{2 - 2\rho} > 0.$$

This completes the proof for this case.

Case 2: $\rho \in (\underline{\rho}, \bar{\rho})$. First, notice that $\rho < \bar{\rho}$ implies that $\tau_{\text{rcv}}^{ac}(g, \cdot, t^C) < \left(\frac{\gamma}{\alpha} - 1\right)(1 - \rho)$ and $\rho > \underline{\rho}$ implies that $\tau_{\text{rcv}}^{ac}(t^A, p_b, t^C) > 0$. Hence, when $\rho \in (1 - \alpha/\gamma, \bar{\rho})$, which implies $\left(\frac{\gamma}{\alpha} - 1\right)(1 - \rho) =$

$\tau^{ac}_{\mathrm{rcv}}(t^A, \cdot, t^C)$, the equilibrium conditions (56) become

$$
\begin{cases}
\tau^{ac}_{\mathrm{rcv}}(t^A, t^B, t^C) \leq 1 - \alpha - \alpha u + (2\alpha - 1)\rho \\[2mm]
\max\{(2\alpha - 1)(1 - \rho), -\tau^{bc}_{\mathrm{rcv}}(t^A, t^B, t^C)\} \leq \frac{1-u}{2}.
\end{cases}
$$

Notice that we can write

$$
\gamma(1 - \rho) + \rho - 1 + \alpha - (2\alpha - 1)(1 - \rho) = \alpha - (1 - \rho)(2\alpha - \gamma) > \alpha - \frac{\alpha}{\gamma}(2\alpha - \gamma) \propto \gamma - \alpha > 0
$$

which implies $(2\alpha - 1)(1 - \rho) < -\tau^{bc}_{\mathrm{rcv}}(\cdot, t^B, t^C)$. As a consequence the equilibrium conditions (56) simplify to (after rearranging the first condition)

$$
\begin{cases}
\left(\frac{\gamma}{2\alpha^2} + 1 - \frac{1}{\alpha}\right)(1 - \rho) \leq \frac{1-u}{2} \\[2mm]
\min\left\{\left(\frac{\gamma}{1-a} - 1\right)(1 - \rho), \alpha - (1 - \gamma)(1 - \rho)\right\} \leq \frac{1-u}{2}
\end{cases}
$$

Notice that

$$
\frac{\gamma}{2\alpha^2} + 1 - \frac{1}{\alpha} - \left(\frac{\gamma}{1-\alpha} - 1\right) \propto \frac{\gamma}{2\alpha} + \alpha - 1 - \gamma\frac{\alpha}{1-\alpha} + \alpha = (2\alpha - 1)\left(1 - \frac{\gamma}{2\alpha(1-\alpha)}\frac{\alpha + 2\alpha^2 - 1}{2\alpha - 1}\right) < 0.
$$

and

$$
\left(\frac{\gamma}{2\alpha^2} + 1 - \frac{1}{\alpha}\right)(1 - \rho) + (1 - \gamma)(1 - \rho) - \alpha < \frac{\alpha}{\gamma}\left(\frac{\gamma}{2\alpha^2} + 2 - \gamma - \frac{1}{\alpha}\right) - \alpha
$$

$$
= \frac{1}{2\alpha} + \frac{2\alpha - 1}{\gamma} - 2\alpha < \frac{1}{2\alpha} + \frac{2\alpha - 1}{\alpha} - 2\alpha = 2 - \frac{1}{2\alpha} - 2\alpha < 0
$$

We then conclude that we need $\alpha < \alpha^{\mathrm{rcv}}_3$. When instead $\rho \in (\underline{\rho}, 1 - \alpha/\gamma]$, the same steps of Case 1 yield condition $\alpha \leq \alpha^{\mathrm{rcv}}_4$.

Case 3: $\rho \geq \overline{\rho}$. In this case, we have $\tau^{ac}_{\mathrm{rcv}}(g, \cdot, t^C) \geq \left(\frac{\gamma}{\alpha} - 1\right)(1 - \rho) = \tau^{ac}_{\mathrm{rcv}}(t^A, \cdot, t^C)$, which implies that the first component of (56) holds. Moreover, since $\rho \geq \overline{\rho}$ implies:

$$
\gamma(1 - \rho) + \rho - (1 - \alpha) - (2\alpha - 1)(1 - \rho) = \alpha - (1 - \rho)(2\alpha - \gamma) \geq (1 - \rho)\left(\gamma\frac{1 - \alpha}{1 - u} - 2\alpha + \gamma\right) > 0,
$$

we also have $-\tau^{bc}_{\mathrm{rcv}}(\cdot, t^B, t^C) \geq (2\alpha - 1)(1 - \rho)$. Hence, the second component of (56) becomes

57

$-\tau^{bc}_{\text{rcv}}(\cdot, t^B, t^C) \leq \max\left\{\frac{1-u}{2}, \tau^{ac}_{\text{rcv}}(g, \cdot, t^C)\right\}$, or $\alpha \leq \alpha^{\text{rcv}}_2$. This completes the proof. $\qquad\square$

To complete the characterization, we can define

$$
\alpha^{\text{rcv}} \equiv
\begin{cases}
\alpha^{\text{rcv}}_1 & \rho \leq \underline{\rho} \\[2mm]
\alpha^{\text{rcv}}_4 & \rho \in (\underline{\rho}, 1 - \alpha/\gamma] \\[2mm]
\alpha^{\text{rcv}}_3 & \rho \in (1 - \alpha/\gamma, \overline{\rho}] \\[2mm]
\alpha^{\text{rcv}}_2 & \rho > \overline{\rho}.
\end{cases}
\tag{58}
$$

We conclude with proposition's final claim: that $\max\{1/2, \alpha^{\text{rcv}}\} \geq \max\{1/2, \alpha^{\text{plu}}\}$. To assist the reader, we begin by rewriting the relevant quantities:

$$
\alpha^{\text{plu}} = \max\left\{\frac{1 - u + 2(1 - \gamma)(1 - \rho)}{1 - u + 2(1 - \rho)}, \frac{1 - u}{1 - u + \gamma(1 - \rho)}\right\}
$$

$$
\alpha^{\text{rcv}}_1 = \frac{1 - u + 2(1 - \rho)}{4(1 - \rho)}
$$

$$
\alpha^{\text{rcv}}_2 = \max\left\{\frac{1 - u + 2(1 - \gamma)(1 - \rho)}{1 - u + 2(1 - \rho)}, \frac{1 - u}{1 - u + \gamma(1 - \rho)}, \frac{1 - u + 2(1 - \gamma)(1 - \rho)}{2}\right\}
$$

$$
\alpha^{\text{rcv}}_3 = \max\left\{\frac{1 - u + 2(1 - \gamma)(1 - \rho)}{1 - u + 2(1 - \rho)}, \frac{1 - u + 2(1 - \gamma)(1 - \rho)}{2}\right\}
$$

$$
\alpha^{\text{rcv}}_4 = \min\left\{\alpha^{\text{rcv}}_1, \frac{1 - u + 2(1 - \gamma)(1 - \rho)}{2}, \frac{1}{2} + \frac{2(1 - \gamma)(1 - \rho) - u}{2u - 4\rho}\right\}
$$

We have $\alpha^{\text{rcv}}_2 \geq \alpha^{\text{plu}}$ by inspection. The remainder of the proof proceeds in three steps.

Step 1. $\alpha^{rcv}_1 > \alpha^{plu}$. This follows from

$$
\frac{1 - u + 2(1 - \rho)}{4(1 - \rho)} - \frac{1 - u + 2(1 - \gamma)(1 - \rho)}{1 - u + 2(1 - \rho)}
$$

$$
\propto (1 - u)^2 + 4\gamma(1 - \rho)^2 - 8(1 - \gamma)(1 - \rho)^2 = (1 - u)^2 + 4\gamma(1 - \rho)^2(1 - 2(1 - \gamma)) > 0
$$

and

$$
\frac{1 - u + 2(1 - \rho)}{4(1 - \rho)} - \frac{1 - u}{1 - u + \gamma(1 - \rho)}
$$

$$
\propto (1 - u)^2 + 2\gamma(1 - \rho)^2 + (2 + \gamma)(1 - \rho)(1 - u) - 4(1 - \gamma)(1 - \rho)
$$

$$\propto \frac{1-u}{1-\rho} + 2\gamma\frac{1-\rho}{1-u} + \gamma - 2 > \frac{1-u}{1-\rho} + \left(\frac{1-u}{1-\rho}\right)^{-1} - \frac{3}{2} > 2 - \frac{3}{2} > 0.$$

Step 2. $\max\{\alpha_3^{rcv}, 1/2\} \geq \max\{\alpha^{plu}, 1/2\}$. Since they share a term in common, to prove this claim it is sufficient to show that $\alpha_3^{rcv} < \alpha^{plu}$ implies that $\max\{\alpha_3^{rcv}, 1/2\} = \max\{\alpha^{plu}, 1/2\}$. To see that, notice that $\alpha_3^{rcv} < \alpha^{plu}$ requires

$$\frac{1-u+2(1-\gamma)(1-\rho)}{1-u+2(1-\rho)} - \frac{1-u}{1-u+\gamma(1-\rho)} < 0$$

$$\Leftrightarrow (1-u)^2 + (1-u)(1-\rho)(2-\gamma) + 2(1-\rho)^2\gamma(1-\gamma) < (1-u)^2 + (1-u)(1-\rho)2$$

$$\Leftrightarrow 2(1-\rho)(1-\gamma) < 1-u \Leftrightarrow \gamma(1-\rho) \geq \frac{1+u-2\rho}{2}.$$

This implies that whenever $\alpha_3^{rcv} < \alpha^{plu}$,

$$\frac{1-u}{1-u+\gamma(1-\rho)} < \frac{1-u}{1-u+\frac{1+u-2\rho}{2}} = \frac{2-2u}{3-u-2\rho} < \frac{2-2u}{3-u-u} < \frac{2-2\frac{1}{2}}{3-1} = \frac{1}{2}.$$

where the the last two inequalities follow from $\rho < u/2$ and $u > 1/2$. But this implies that whenever $\alpha_3^{rcv} < \alpha^{plu}$:

$$\max\left\{\alpha^{plu}, \frac{1}{2}\right\} = \frac{1}{2} \leq \max\left\{\alpha_3^{rcv}, \frac{1}{2}\right\}.$$

Step 3. When $\rho \in (\underline{\rho}^{rcv}, 1 - \alpha/\gamma]$ and $\max\{\alpha_4^{rcv}, 1/2\} \geq \max\{\alpha^{plu}, 1/2\}$. First, from Step 1, we know that $\alpha_1^{rcv} \geq \alpha^{plu}$. Second, from Step 2 we know that $\alpha^{plu} > \frac{1}{2}$ requires

$$\alpha_2^{plu} = \frac{1-u+2(1-\gamma)(1-\rho)}{1-u+2(1-\rho)}$$

and $1 - u > 2(1-\rho)(2\gamma - 1)$. Finally, we argue that $1 - u > 2(1-\rho)(2\gamma - 1)$ implies

$$\min\left\{\frac{1-u+2(1-\gamma)(1-\rho)}{2}, \frac{1}{2} + \frac{2(1-\gamma)(1-\rho)-u}{2(u-2\rho)}\right\}$$
$$= \frac{1-u+2(1-\gamma)(1-\rho)}{2} > \frac{1-u+2(1-\gamma)(1-\rho)}{1-u+2(1-\rho)},$$

where, using $\rho \le 1 - \alpha/\gamma \Leftrightarrow (1-\rho)\gamma \ge \alpha$ the inequality follows from

$$1 - u + 2(1-\rho) > (1-\rho)(4\gamma - 2 + 2) \ge 4\alpha > 2,$$

and the first equality, using $\rho > \underline{\rho} \Leftrightarrow (1-\rho) < \frac{1-\alpha}{1-\gamma}$, follows from

$$u - 2\rho < 1 - 2(1-\rho)(2\gamma - 1) - 2\rho = 4(1-\gamma)(1-2\rho) - 1 < 4(1-\alpha) - 1 \le 1.$$

This completes the proof. $\square$

Proof of Proposition 6. It is easy to verify that our earlier analysis ensures that a's and b's strategies are best responses, given $p_c = t^C$. We therefore focus on verifying that c does not have a profitable deviation from $p_c = t^C$ to $p_c = g$. Recall that for $I \in \{A, B\}$ and any (p_a, p_b), $\tau_{ac}^I(p_a, t^C) < 0$ and $\tau_{bc}^I(p_b, t^C) > 0$.

Step 1: $\alpha \ge \alpha^{rcv}$. Fix $p_a = t^A$ and $p_b = g$. Candidate c wins the election with strategy $p_c = t^C$ if and only if

$$\min\left\{\frac{u-1}{2}, u - \gamma - \rho(1-\gamma)\right\} \le \tau \le \frac{\gamma - \alpha}{\alpha}(1-\rho).$$

Suppose candidate c instead selects $p_c = g$. Candidate b wins second preferences from group-A voters for all $\tau \le \theta$. This yields that candidate c continues to lose the election when $\tau \le \min\left\{\frac{u-1}{2}, u - \gamma - \rho(1-\gamma)\right\}$. We further claim that there exists $\hat\tau < \frac{\gamma - \alpha}{\alpha}(1-\rho)$ such that c loses whenever $\tau > \hat\tau$ after she deviates to $p_b = g$. To verify this claim, recognize that for any $\tau > 0$, candidate c's total first and second preferences are *at most* $\gamma\phi(u-\rho) + (1-\alpha)\phi(u - \theta - \rho)$, while candidate a's total first and second preferences are no less than $\alpha\phi(1+\tau)$, which strictly increases with τ. Recalling that

$$\alpha\phi\left(1 + \frac{\gamma - \alpha}{\alpha}(1-\rho)\right) = \gamma\phi(1-\rho)$$

it is therefore sufficient to show that

$$\gamma\phi(u-\rho) + (1-\alpha)\phi(u - \theta - \rho) < \alpha\phi\left(1 + \frac{\gamma - \alpha}{\alpha}(1-\rho)\right)$$

$$\Longleftrightarrow (1-\alpha)(u - \theta - \rho) < \gamma(1-u).$$

This holds for all $\theta > u/2$ and $\rho > \overline{\rho}$ if

$$\gamma(1 - u) > (1 - \alpha)\left(\frac{u}{2} - 1 + \frac{\alpha}{1 - \alpha}\frac{1 - u}{\gamma}\right).$$

The difference of the LHS and the RHS is linear in u. When $u = 1$, the inequality trivially holds. When $u = \alpha$, we need to verify that $\gamma > \frac{\alpha}{2} - 1 + \frac{\alpha}{\gamma}$, which is true because $\alpha < \gamma$. $\square$

Step 2: $\alpha \leq \alpha^{rcv}$. Fix $p_a = t^A$ and $p_b = g$. If candidate c chooses $p_c = t^C$, she wins whenever

$$\tau \in \left[\max\left\{\frac{1 - \alpha - \gamma}{1 - \alpha}(1 - \rho), 1 - \alpha - \gamma - \rho(1 - \gamma)\right\}, \frac{\gamma - \alpha}{\alpha}(1 - \rho)\right].$$

We already showed that if c instead chooses $p_c = g$, then she loses for any $\tau \geq \frac{\gamma - \alpha}{\alpha}(1 - \rho)$ when $\rho \geq \overline{\rho}$. To rule out a profitable deviation for c, it is therefore sufficient to verify that if c chooses $p_c = g$, she loses for all $\tau \leq \max\left\{\frac{1 - \alpha - \gamma}{1 - \alpha}(1 - \rho), 1 - \alpha - \gamma - \rho(1 - \gamma)\right\}$. We consider two possible cases.

Case 1. Suppose, first, $\frac{1 - \alpha - \gamma}{1 - \alpha}(1 - \rho) \geq 1 - \alpha - \gamma - \rho(1 - \gamma)$. By a similar argument to Step 1, when $p = (t^A, t^B, g)$ a sufficient condition that candidate c loses for all $\tau \leq \frac{1 - \alpha - \gamma}{1 - \alpha}(1 - \rho)$ is:

$$\gamma(u - \rho) + \alpha(u - \theta - \rho) \leq \gamma(1 - \rho).$$

Since $\theta \geq u/2$, this is true if $\rho \geq \frac{u}{2} - \frac{\gamma}{\alpha}(1 - u)$; since $\rho \geq \overline{\rho}$, this is true if $\overline{\rho} > \frac{u}{2} - \frac{\gamma}{\alpha}(1 - u)$, i.e., if

$$1 - \frac{u}{2} > (1 - u)\left(\frac{\alpha}{\gamma(1 - \alpha)} - \frac{\gamma}{\alpha}\right).$$

The difference is linear in u, and trivially holds if $u = 1$. Since $u \geq \alpha$, it is sufficient to show that

$$1 - \frac{\alpha}{2} > \frac{\alpha}{\gamma} - \frac{1 - \alpha}{\alpha}\gamma. \tag{59}$$

This constraint eases as γ increases. Setting $\gamma = \alpha$, it is sufficient to verify that $\alpha \leq \frac{2}{3}$. Suppose, to the contrary, $\alpha > \frac{2}{3}$. Since we also have $\gamma > \alpha$, this implies $\gamma > \frac{2}{3}$. This implies $\frac{1 - \alpha - \gamma}{1 - \alpha}(1 - \rho) < \frac{\frac{1}{3} - \frac{2}{3}}{\frac{1}{3}}(1 - \rho) = \rho - 1 < \frac{u}{2} - 1 < -\frac{1}{2}$, contradicting $\alpha \leq \alpha^{rcv}$. We conclude that we must have $\alpha \leq \frac{2}{3}$ if $\frac{1 - \alpha - \gamma}{1 - \alpha}(1 - \rho) \geq 1 - \alpha - \gamma - \rho(1 - \gamma)$ and $\alpha \leq \alpha^{rcv}$, which implies that (59) is satisfied, and concludes this case.

Case 2. Suppose, instead, $1 - \alpha - \gamma - \rho(1 - \gamma) > \frac{1-\alpha-\gamma}{1-\alpha}(1 - \rho)$. This is consistent with $\alpha \leq \alpha^{\mathrm{rcv}}$ only if $1 - \alpha - \gamma - \rho(1 - \gamma) \geq (u - 1)/2$. Notice that if the strategy profile is (t^A, t^B, g), c wins fewer first preferences than candidate a for all τ such that

$$\alpha F(1 + \tau) \geq \gamma F(u) \iff \tau \geq \rho - 1 + \frac{\gamma}{\alpha}(u - \rho) = \hat{\tau}.$$

We claim that $\hat{\tau} < (u - 1)/2$. This follows because $\hat{\tau}$ strictly decreases in ρ, and thus

$$\hat{\tau} - (u - 1)/2 < \overline{\rho} - 1 + \frac{\gamma}{\alpha}(u - \overline{\rho}) - (u - 1)/2 = -\frac{(1 - u)\left(\alpha^2(\gamma + 2) - \alpha\gamma(2\gamma + 3) + 2\gamma^2\right)}{2(1 - \alpha)\alpha\gamma}.$$

Straightforward computation yields that $\alpha^2(\gamma + 2) - \alpha\gamma(2\gamma + 3) + 2\gamma^2 > 0$ for all $\alpha \leq \gamma \leq \frac{3}{5}$. To verify that $\gamma \leq \frac{3}{5}$, recall that $1 - \alpha - \gamma - \rho(1 - \gamma) \geq (u - 1)/2$, which requires $1 - \alpha - \gamma - \overline{\rho}(1 - \gamma) \geq (u - 1)/2$, which is equivalent to

$$1 - \alpha - \gamma - (1 - \gamma)\left(1 - \frac{\alpha}{1 - \alpha}\frac{1 - u}{\gamma}\right) - \frac{u - 1}{2} \geq 0,$$

which is true for $u \geq \alpha$ only if

$$1 - \alpha - \gamma - (1 - \gamma)\left(1 - \frac{\alpha}{\gamma}\right) - \frac{\alpha - 1}{2} \geq 0 \iff \gamma \leq \frac{2\alpha}{5\alpha - 1},$$

and combining with the requirement $\alpha \leq \gamma$, this implies $\gamma \leq \frac{3}{5}$.

We further claim that c's combined first and second preferences are strictly less than b's first preferences for any $\tau \leq \hat{\tau}$. A sufficient condition for this to be true for any $\theta \geq u/2$ is that

$$\gamma\phi(u - \rho) + \alpha\phi(u/2 - \rho) < (1 - \alpha)\phi(1 - \hat{\tau} - \rho) \iff \rho > \frac{\alpha^2(u + 4) - 4\alpha + 2\gamma u}{2(\alpha(3\alpha - 2) + \gamma)}.$$

It is sufficient, then, to show that

$$\overline{\rho} - \frac{\alpha^2(u + 4) - 4\alpha + 2\gamma u}{2(\alpha(3\alpha - 2) + \gamma)} > 0 \iff 1 - \frac{\alpha^2(u + 4) - 4\alpha + 2\gamma u}{2(\alpha(3\alpha - 2) + \gamma)} - \frac{\alpha(1 - u)}{(1 - \alpha)\gamma} > 0.$$

This difference is linear in u: it is satisfied for $u = 1$, and further holds for $u = \alpha$ if

$$1 - \frac{\alpha}{\gamma} - \frac{\alpha\left(\alpha^2 + 4\alpha + 2\gamma - 4\right)}{2(\alpha(3\alpha - 2) + \gamma)} > 0.$$

Since the LHS strictly decreases in $\alpha \le \gamma$, it is sufficient to verify that the inequality holds if $\alpha = \gamma$, which requires that $\gamma \le \sqrt{13} - 3$, which follows from $\alpha \le \frac{3}{5}$. $\square$

Proof of Propositions 7 and 8. We proceed by cases.

If $\rho > \bar{\rho}$ and $\alpha \ge \alpha^{\mathrm{rcv}}$, the unique equilibrium under both plurality and RCV is (t^A, g, t^C). Under both rules, a wins if and only if $\tau \ge \frac{\gamma - \alpha}{\alpha}(1 - \rho)$, b wins if and only if $\tau \le \frac{u-1}{2}$. As a consequence, the unique equilibrium under each system induces the same outcome.

If $\rho > \bar{\rho}$ and $\alpha \in (\alpha^{\mathrm{plu}}, \alpha^{\mathrm{rcv}})$ the unique equilibrium under plurality is (t^A, g, t^C), but the unique equilibrium under RCV is (t^A, t^B, t^C). Since $\alpha < \alpha^{\mathrm{rcv}}$, b's probability of winning is strictly higher under RCV and c's probability of winning is strictly lower under RCV, while a's probability of winning is the same under both systems.

If $\rho > \bar{\rho}$ and $\alpha < \alpha^{\mathrm{plu}}$, the unique equilibrium under both plurality and RCV is (t^A, t^B, t^C). Under plurality, a wins if and only if $\tau \ge \frac{\gamma - \alpha}{\gamma}(1 - \rho)$, and b wins if and only if $\tau \le \frac{1 - \alpha - \gamma}{1 - \alpha}(1 - \rho)$; otherwise, c wins. Under RCV, a wins if and only if $\tau \ge \frac{\gamma - \alpha}{\gamma}(1 - \rho)$, and b wins if and only if $\tau \le \max\left\{\frac{1 - \alpha - \gamma}{1 - \gamma}(1 - \rho), 1 - \alpha - \gamma - \rho(1 - \gamma)\right\}$. So, b's probability of winning is weakly higher under RCV and c's probability of winning is weakly lower under RCV, while a's probability of winning is the same under both systems.

If $\rho < \underline{\rho}$ and $\alpha > \alpha^{\mathrm{rcv}}$, plurality's unique equilibrium outcome is that a wins if $\tau > 0$, and b wins if $\tau < 0$, while RCV's unique equilibrium is (t^A, g, t^C). Under plurality, candidate c therefore wins with probability zero. Under RCV, a wins if $\tau \ge \gamma - \alpha + \rho(1 - \gamma)$, while b wins if $\tau \le \frac{u-1}{2}$. We conclude that a's probability of winning is strictly higher under RCV, that b's probability of winning is strictly lower under RCV, and that c's probability of winning is strictly higher under RCV.

If $\rho < \underline{\rho}$ and $\alpha \in (\alpha^{\mathrm{plu}}, \alpha^{\mathrm{rcv}})$, plurality's unique equilibrium outcome is that a wins if $\tau > 0$, and b wins if $\tau < 0$, while RCV's unique equilibrium is (t^A, t^B, t^C). Under RCV, a wins if $\tau \ge \max\{(1 - 2\alpha)(1 - \rho), \gamma - \alpha + \rho(1 - \gamma)\} < 0$, and b wins if $\tau \le \min\{(1 - 2\alpha)(1 - \rho), 1 - \alpha - $

$\gamma - \rho(1-\gamma)\} < 0$. We conclude that a's probability of winning is strictly higher under RCV, that b's probability of winning is strictly lower under RCV, and that c's probability of winning is weakly higher under RCV. $\square$