The Equivalence Principle

Claus Lämmerzahl

Centre for Applied Space Technology and Microgravity (ZARM),
University of Bremen, 28359 Bremen, Germany

MICROSCOPE Colloquium
Paris, September 19, 2011

Outline

1. Introduction: The Equivalence Principle(s)
   - Geometry
   - The present situation
   - Need for Quantum Gravity

Outline

Outline

1. Introduction: The Equivalence Principle(s)
   - Geometry
   - The present situation
   - Need for Quantum Gravity
2. General theoretical remarks
   - General formalism
   - The importance of UFF: Schiff's conjecture
3. Further aspects
   - Particles with degrees of freedom
   - Universality of the gravitational field
   - Quantum tests

Outline

1. Introduction: The Equivalence Principle(s)
   - Geometry
   - The present situation
   - Need for Quantum Gravity
2. General theoretical remarks
   - General formalism
   - The importance of UFF: Schiff's conjecture
3. Further aspects
   - Particles with degrees of freedom
   - Universality of the gravitational field
   - Quantum tests
4. Summary

Outline

Outline

Geometrization

Most important formula for physics and mathematics

$$m_{\mathrm{i}}\ddot{\boldsymbol{x}} = \boldsymbol{F}$$

Model for force

$$\boldsymbol{F} = -m_{\mathrm{g}}\boldsymbol{\nabla}U \qquad U = \text{Newton potential}$$

Acceleration

$$\boldsymbol{a} = \ddot{\boldsymbol{x}} = -\frac{m_{\mathrm{g}}}{m_{\mathrm{i}}}\boldsymbol{\nabla}U$$

Experiment

$$\eta = \frac{a_2 - a_1}{\frac{1}{2}(a_1 + a_1)} = \frac{(m_{\mathrm{g}}/m_{\mathrm{i}})_2 - (m_{\mathrm{g}}/m_{\mathrm{i}})_1}{\frac{1}{2}\left((m_{\mathrm{g}}/m_{\mathrm{i}})_2 + (m_{\mathrm{g}}/m_{\mathrm{i}})_1\right)} \leq 2 \cdot 10^{-13}$$

Idealization: Equivalence Principle $m_{\mathrm{g}} = m_{\mathrm{i}}$
$\Rightarrow$ path does not depend on particle $\leftrightarrow$ geometry

Equivalence Principle = geometrization of gravitational interaction

Universality principles

- Gravity acts on all kinds of matter
- Gravity acts on all kinds of matter in the same way
- Gravity acts on all kinds of clocks
- Gravity acts on all kinds of clocks in the same way
- Gravity is created from all kinds of matter
- Gravity is created from all kinds of matter in the same way

Universality principles

- Gravity acts on all kinds of matter
- Gravity acts on all kinds of matter in the same way
- Gravity acts on all kinds of clocks
- Gravity acts on all kinds of clocks in the same way
- Gravity is created from all kinds of matter
- Gravity is created from all kinds of matter in the same way

Universality principles

- Gravity acts on all kinds of matter
- Gravity acts on all kinds of matter in the same way
- Gravity acts on all kinds of clocks
- Gravity acts on all kinds of clocks in the same way
- Gravity is created from all kinds of matter
- Gravity is created from all kinds of matter in the same way

Universality principles

- Gravity acts on all kinds of matter
- Gravity acts on all kinds of matter in the same way
- Gravity acts on all kinds of clocks
- Gravity acts on all kinds of clocks in the same way
- Gravity is created from all kinds of matter
- Gravity is created from all kinds of matter in the same way

Universality principles

- Gravity acts on all kinds of matter
- Gravity acts on all kinds of matter in the same way
- Gravity acts on all kinds of clocks
- Gravity acts on all kinds of clocks in the same way
- Gravity is created from all kinds of matter
- Gravity is created from all kinds of matter in the same way

Universality principles

- Gravity acts on all kinds of matter
- Gravity acts on all kinds of matter in the same way
- Gravity acts on all kinds of clocks
- Gravity acts on all kinds of clocks in the same way
- Gravity is created from all kinds of matter
- Gravity is created from all kinds of matter in the same way

Universality principles

- Gravity acts on all kinds of matter
- Gravity acts on all kinds of matter in the same way
- Gravity acts on all kinds of clocks
- Gravity acts on all kinds of clocks in the same way
- Gravity is created from all kinds of matter
- Gravity is created from all kinds of matter in the same way

Universality principles

- Gravity acts on all kinds of matter
- Gravity acts on all kinds of matter in the same way
- Gravity acts on all kinds of clocks
- Gravity acts on all kinds of clocks in the same way
- Gravity is created from all kinds of matter
- Gravity is created from all kinds of matter in the same way

$$\boxed{\text{gravity} \quad \Leftrightarrow \quad \text{universality principles} \quad \Leftrightarrow \quad \text{gravity} = \text{geometry}}$$

Universality principles

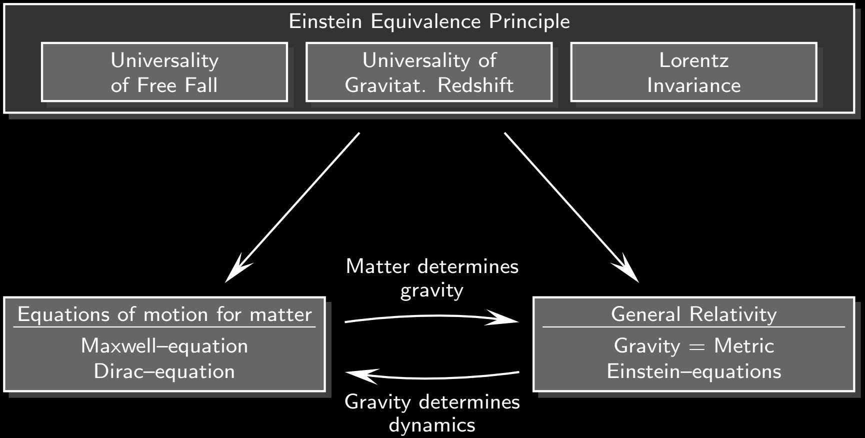
- Gravity acts on all kinds of matter
- Gravity acts on all kinds of matter in the same way
- Gravity acts on all kinds of clocks
- Gravity acts on all kinds of clocks in the same way
- Gravity is created from all kinds of matter
- Gravity is created from all kinds of matter in the same way

$$\boxed{\text{gravity} \quad \Leftrightarrow \quad \text{universality principles} \quad \Leftrightarrow \quad \text{gravity} = \text{geometry}}$$

It is a miracle that these universality principles hold with the present high experimental accuracy

Outline

Structure of standard physics



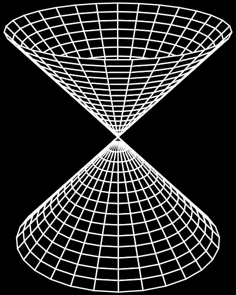
The present situation

All aspects of Lorentz invariance are experimentally well tested and confirmed

Foundations

Postulates

- $c = const$
- Principle of Relativity



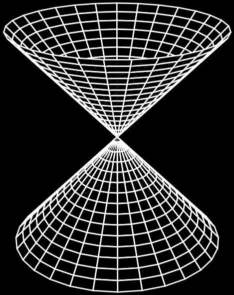
The present situation

All aspects of Lorentz invariance are experimentally well tested and confirmed

Foundations

Postulates

- $c = const$
- Principle of Relativity



Tests

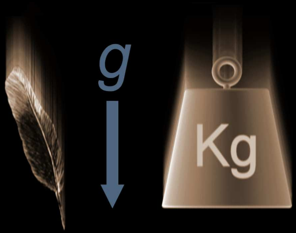
- Independence of c from velocity of the source
- Universality of c
- Isotropy of c
- Independence of c from velocity of the laboratory
- Time dilation
- Isotropy of physics (Hughes–Drever experiments)
- Independence of physics from the velocity of the laboratory

The present situation

Many aspects of the Universality of Free Fall are experimentally well tested and confirmed

Postulate

In a gravitational field all structureless test particles fall in the same way

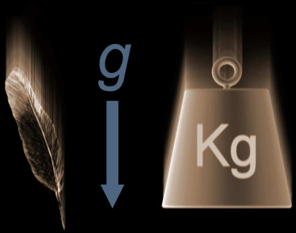


The present situation

Many aspects of the Universality of Free Fall are experimentally well tested and confirmed

Postulate

In a gravitational field all structureless test particles fall in the same way



Tests

UFF for

- Neutral bulk matter
- Charged particles
- Particles with spin

No test so far for

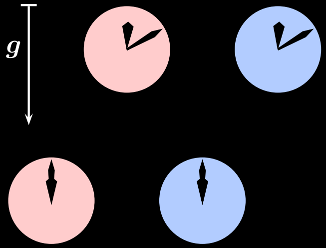
- Anti particles

The present situation

Many aspects of the Universality of the Gravitational Redshift are experimentally well tested and confirmed

Postulate

In a gravitational field all clocks behave in the same way

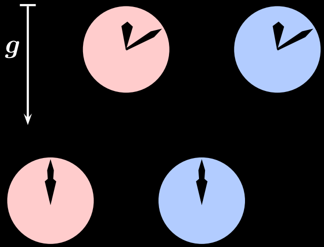


The present situation

Many aspects of the Universality of the Gravitational Redshift are experimentally well tested and confirmed

Postulate

In a gravitational field all clocks behave in the same way



Tests

UGR for

- Atomic clocks: electronic
- Atomic clocks: hyperfine
- Molecular clocks: vibrational
- Molecular clocks: rotational
- Resonators
- Nuclear transitions

No test so far for

- Anti clocks

The present situation

All predictions of General Relativity are experimentally well tested and confirmed

Foundations

The Einstein Equivalence Principle

- Universality of Free Fall
- Universality of Gravitational Redshift
- Local Lorentz Invariance

The present situation

All predictions of General Relativity are experimentally well tested and confirmed

Foundations

The Einstein Equivalence Principle

- Universality of Free Fall
- Universality of Gravitational Redshift
- Local Lorentz Invariance

$\Downarrow$

Implication

Gravity is a metrical theory

Ehlers, Pirani & Schild 1972

The present situation

All predictions of General Relativity are experimentally well tested and confirmed

Foundations

The Einstein Equivalence Principle

- Universality of Free Fall
- Universality of Gravitational Redshift
- Local Lorentz Invariance

$\Downarrow$

Implication

Gravity is a metrical theory

$\Rightarrow$

Predictions for metrical theories

- Solar system effects
  - Perihelion shift
  - Gravitational redshift
  - Deflection of light
  - Gravitational time delay
  - Lense–Thirring effect
  - Schiff effect
- Strong gravitational fields
  - Binary systems
  - Black holes
- Gravitational waves

The present situation

All predictions of General Relativity are experimentally well tested and confirmed

Foundations

The Einstein Equivalence Principle

- Universality of Free Fall
- Universality of Gravitational Redshift
- Local Lorentz Invariance

$\Downarrow$

Implication

Gravity is a metrical theory

$\Rightarrow$

Predictions for metrical theories

- Solar system effects
  - Perihelion shift
  - Gravitational redshift
  - Deflection of light
  - Gravitational time delay
  - Lense–Thirring effect
  - Schiff effect
- Strong gravitational fields
  - Binary systems
  - Black holes
- Gravitational waves

$\Downarrow$

General Relativity

Outline

Quantum gravity

Incompatibilities

- All standard quantization schemes not applicable
- The problem of time

$$\left.\begin{matrix}\text{Time in quantum theory} \\ \text{external variable}\end{matrix}\right\} \quad \overset{\text{incompatible}}{\longleftrightarrow} \quad \left\{\begin{matrix}\text{Time in General Relativity} \\ \text{dynamical variable}\end{matrix}\right.$$

Quantum gravity

Incompatibilities

- All standard quantization schemes not applicable
- The problem of time

$$\left.\begin{matrix}\text{Time in quantum theory}\\ \text{external variable}\end{matrix}\right\} \quad \overset{\text{incompatible}}{\longleftrightarrow} \quad \left\{\begin{matrix}\text{Time in General Relativity}\\ \text{dynamical variable}\end{matrix}\right.$$

Further reasons for a need of Quantum Gravity

- If matter is quantized, then the interaction has to be quantized, too (Bohr, Rosenfeld)
- Singularities – black holes
  - Classical GR: singularity theorems
  - Quantization circumvents breakdown of physics in the early universe and in black holes

The description of physics is not yet complete

Today's standard theories and standard space–time notion as explored by point particles, light rays, and fields

Frame theories	**Interactions**
Quantum theory	Electrodynamics
Special Relativity	Gravity
General Relativity	Weak interaction
Statistical mechanics	Strong interaction
Problems	**Wish**
• Incompatibility of quantum theory and General Relativity • Problem of time • Occurrence of singularities	Unification of all interactions

Need of modifications of standard theories, but standard theories derived from observations

$\Rightarrow$ need for more precise measurements, other observations

Implications of a new theory

Unresolved fundamental inconsistency

$\Rightarrow$ Standard physics cannot be completely correct

$\Rightarrow$ There have to be modifications to standard physics

Modifications on the effective level

- $\Rightarrow$ Modifications in Maxwell, Dirac, Einstein equations
- $\Rightarrow$ Violation of Einstein Equivalence Principle
- $\Rightarrow$ Search for violations of the Einstein Equivalence Principle

Implications of a new theory

Unresolved fundamental inconsistency
$\Rightarrow$ Standard physics cannot be completely correct
$\Rightarrow$ There have to be modifications to standard physics

Modifications on the effective level

- $\Rightarrow$ Modifications in Maxwell, Dirac, Einstein equations
- $\Rightarrow$ Violation of Einstein Equivalence Principle
- $\Rightarrow$ Search for violations of the Einstein Equivalence Principle

Modifications on the quantum level

Modified notion of space–time (e.g. space–time fluctuations)
But: space-time is explored by particles, photons, ...

- $\Rightarrow$ Modified space–time properties result in modified equations of motion $\Rightarrow$ Search for violation of the Einstein Equivalence Principle
- $\Rightarrow$ Search for fundamental noise, decoherence, non–conserved probablility, ...

Implications of a new theory

Unresolved fundamental inconsistency

$\Rightarrow$ Standard physics cannot be completely correct

$\Rightarrow$ There have to be modifications to standard physics

Modifications on the effective level

- $\Rightarrow$ Modifications in Maxwell, Dirac, Einstein equations
- $\Rightarrow$ Violation of Einstein Equivalence Principle
- $\Rightarrow$ Search for violations of the Einstein Equivalence Principle

Modifications on the quantum level

Modified notion of space–time (e.g. space–time fluctuations)
But: space-time is explored by particles, photons, ...

- $\Rightarrow$ Modified space–time properties result in modified equations of motion $\Rightarrow$ Search for violation of the Einstein Equivalence Principle
- $\Rightarrow$ Search for fundamental noise, decoherence, non–conserved probablility, ...

Modifications certainly show up in a violation of UFF

Outline

Outline

1. Introduction: The Equivalence Principle(s)
   - Geometry
   - The present situation
   - Need for Quantum Gravity
2. General theoretical remarks
   - General formalism
   - The importance of UFF: Schiff's conjecture
3. Further aspects
   - Particles with degrees of freedom
   - Universality of the gravitational field
   - Quantum tests
4. Summary

Description of tests of the universality principles

Purpose: parametrization of deviations, comparison of different experiments

Haugan formalism (Haugan, AP 1979)

Ansatz: effective atomic Hamiltonian (can be derived from modified Dirac and modified Maxwell, from $TH\epsilon\mu$–formalism, ...)

$$H = mc^2 + \frac{1}{2m}\left(\delta^{ij} + \frac{\delta m_{\mathrm{i}}^{ij}}{m}\right) p_i p_j + m\left(\delta_{ij} + \frac{\delta m_{\mathrm{g}ij}}{m}\right) U^{ij}(\boldsymbol{x}) + \ldots$$

with

$$\delta m_{\mathrm{i}ij} \quad = \quad \text{anomalous inertial mass tensor (depends on atom and state)}$$
$$\delta m_{\mathrm{g}ij} \quad = \quad \text{anomalous gravitational mass tensor ('')}$$
$$U^{ij} \quad = \quad G \int \frac{(\boldsymbol{x}-\boldsymbol{x}')^i (\boldsymbol{x}-\boldsymbol{x}')^j \rho(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|^3} d^3x' \,, \qquad \delta_{ij} U^{ij} = U$$

Description of tests of the universality principles

Universality of Free Fall

- Acceleration

$$a^i = \delta^{ij}\partial_j U(\boldsymbol{x}) + \frac{\delta m_{\rm i}^{ij}}{m}\partial_j U(\boldsymbol{x}) + \delta^{ij}\frac{\delta m_{{\rm g}kl}}{m}\partial_j U^{kl}(\boldsymbol{x})$$

- For diagonal mass tensors $\delta m_{{\rm i}ij} = \delta m_{\rm i}\delta_{ij}$, $\delta m_{{\rm g}ij} = \delta m_{\rm g}\delta_{ij}$:

$$a^i = \delta^{ij}\frac{m_{\rm g}}{m_{\rm i}}\partial_j U$$

- Comparison of acceleration of two different particles: Eötvös coefficient

$$\eta = \frac{a_2 - a_1}{\frac{1}{2}\left(a_2 + a_1\right)} = \frac{\left(m_{\rm g}/m_{\rm i}\right)_2 - \left(m_{\rm g}/m_{\rm i}\right)_1}{\frac{1}{2}\left(\left(m_{\rm g}/m_{\rm i}\right)_2 + \left(m_{\rm g}/m_{\rm i}\right)_1\right)}$$

Tests of UFF

1 Tests with bulk matter

Method	Grav field	Accuracy	Experiment
Torsion pendulum	Sun	$\eta \leq 2 \cdot 10^{-13}$	Adelberger 2006

2 Tests with quantum matter

Method	Grav field	Accuracy	Experiment
Atom interferometry	Earth	$\eta \leq 10^{-9}$	Chu, Peters 1999
		$\eta \leq 10^{-6}$	Fray et al 2004

3 Gravitational self energy

Method	Grav field	Accuracy	Experiment
Torsion pendulum and LLR	Sun	$\eta \leq 1.3 \cdot 10^{-3}$	Baessler et al 1999

Tests of UFF

4 Charged particles

Method	Grav field	Accuracy	Experiment
Free fall of electron	Earth	$\eta \leq 10^{-1}$	Witteborn & Fairbank 1967

5 Particles with spin

Method	Grav field	Accuracy	Experiment
Weighting polarized bodies	Earth	$\eta \leq 10^{-8}$	Hsie et al 1989

6 Anti–particles

Method	Grav field	Accuracy	Experiment
Free fall of anti–Hydrogen	Earth	$\eta \leq 10^{-3} - 10^{-5}$	(estimate)

Description of tests of the universality principles

Universality of kinetic and Gravitational Redshift

- frequency comparison

$$\frac{\nu_2}{\nu_1} = \frac{\nu_2^0}{\nu_1^0}\left(1 + \underbrace{\left(\frac{\Delta\delta m_{iij}^{(2)}}{m^{(2)}} - \frac{\Delta\delta m_{iij}^{(1)}}{m^{(1)}}\right) v^i v^j}_{\text{frequency ratio depends on velocity}} + \underbrace{\left(\frac{\Delta\delta m_{gij}^{(2)}}{m^{(2)}} - \frac{\Delta\delta m_{gij}^{(1)}}{m^{(1)}}\right) U^{ij}(\boldsymbol{x})}_{\text{frequency ratio depends on position}}\right)$$

- 1st term: singles out a certain frame of reference $\Rightarrow$ violation of Local Lorentz invariance
- 2nd term: singles out a certain position $\Rightarrow$ violation of Local Position invariance $\Leftrightarrow$ violation of Universality of Gravitational Redshift
- for diagonal anomalous mass terms

$$\frac{\nu_2}{\nu_1} = \frac{\nu_2^0}{\nu_1^0}\left(1 + \underbrace{(\alpha_{\text{clock }2} - \alpha_{\text{clock }1})v^2}_{\text{violation of LLI, MS}} + \underbrace{(\beta_{\text{clock }2} - \beta_{\text{clock }1})\, U(\boldsymbol{x})}_{\text{violation of UGR = LPI}}\right)$$

Redshift tests

Tests of the Universality of Kinematical Redshift

no real comparison experiment exists: all experiment just test the kinematical redshift and look for deviations of $|\alpha - 1|$

Method	Accuracy	Experiment
Doppler shift	$1 \cdot 10^{-2}$	Ives & Stilwell 1938
2–photon spectroscopy	$1.4 \cdot 10^{-6}$	Riies et al, PRL 1988
saturation spectroscopy	$8 \cdot 10^{-8}$	Saathoff et al, PRL 2003

Tests of the Universality of Gravitational Redshift

Comparison	Accuracy	Experiment
Cs – Resonator	$2 \cdot 10^{-2}$	Turneaure & Stein 1987
Mg – Cs (fine structure)	$7 \cdot 10^{-4}$	Godone et al 1995
Resonator – I_2 (electronic)	$4 \cdot 10^{-2}$	Braxmaier et al, PRL 2002
Cs – H-Maser (hf)	$2.5 \cdot 10^{-5}$	Bauch et al, PRD 2002
Cs – Hg	$5 \cdot 10^{-6}$	Fortier et al, PRL 2007

Physical systems

- Atomic systems
  - Principal state
  - Fine structure
  - Hyperfine structure
- Molecular systems
  - Rotational dof
  - Vibrational dof
- Light clocks
- Gravitational clocks
  - Planetary motion
  - Binary systems
- Rotation
  - Earth
  - Pulsars
- Decay of particles

All based on different physical principles, laws.

- Systems of different nature exhibit different depend. on fundam. constants
- EEP tests are tests of the coupling of interaction fields (Maxwell, weak, strong) to gravity
- UFF + UGR $\leftrightarrow$ time– and space–dependence of fundamental constants
- UFF more sensitive than UGR (Nordtvedt 2003)

Part of a larger scheme: Hierarchy of theories

Full theory

Quantum Gravity

Experiment Observation

acceleration
clock readout
interference fringes
counting events
...

Part of a larger scheme: Hierarchy of theories

Full theory	Quantum Gravity
↓ derived	
Effective theory	Dilaton scenarios Axion fields Torsion ...
Experiment Observation	acceleration clock readout interference fringes counting events ...

Part of a larger scheme: Hierarchy of theories

Full theory	Quantum Gravity
↓ derived	
Effective theory	Dilaton scenarios Axion fields Torsion ...
Phenomenology Test theory	Standard Model Extension PPN formalism c^2–formalism Robertson–Mansouri–Sexl $TH\epsilon\mu$–formalism $\chi - g$–formalism ...
↕	
Experiment Observation	acceleration clock readout interference fringes counting events ...

Part of a larger scheme: Hierarchy of theories

Full theory	Quantum Gravity
derived $\downarrow$	
Effective theory	Dilaton scenarios Axion fields Torsion ...
$\cap$	
Phenomenology Test theory	Standard Model Extension PPN formalism c^2–formalism Robertson–Mansouri–Sexl $TH\epsilon\mu$–formalism $\chi - g$–formalism ...
$\updownarrow$	
Experiment Observation	acceleration clock readout interference fringes counting events ...

Part of a larger scheme: Hierarchy of theories

Full theory

derived $\downarrow$

Effective theory

$\bigcap$

Phenomenology
Test theory

$\updownarrow$

Experiment
Observation

Quantum Gravity

Dilaton scenarios
Axion fields
Torsion
...

Standard Model Extension
PPN formalism
c^2–formalism
Robertson–Mansouri–Sexl
$TH\epsilon\mu$–formalism
$\chi - g$–formalism
...

acceleration
clock readout
interference fringes
counting events
...

Test theories
mediate between
experiment and full theory

Test theories, Phenomenology

Phenomenology = Generalizations of Maxwell and Dirac equations

$$
\begin{aligned}
4\pi j^\mu &= \eta^{\mu\rho}\eta^{\nu\sigma}\partial_\nu F_{\rho\sigma} \\
0 &= i\gamma^a D_a\psi + m\psi
\end{aligned}
$$

with

$$\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab}$$

- Standard equations

Test theories, Phenomenology

Phenomenology = Generalizations of Maxwell and Dirac equations

$$4\pi j^\mu = \eta^{\mu\rho}\eta^{\nu\sigma}\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\nu\sigma}\partial_\nu F_{\rho\sigma}$$

$$0 = i\gamma^a D_a \psi + m\psi + M\psi$$

with

$$\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab} + X^{ab}$$

- Standard equations
- Particular case: Standard Model Extension

Test theories, Phenomenology

Phenomenology = Generalizations of Maxwell and Dirac equations

$$4\pi j^\mu = \eta^{\mu\rho}\eta^{\nu\sigma}\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\nu\sigma}\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\sigma} F_{\rho\sigma}$$

$$0 = i\gamma^a D_a \psi + m\psi + M\psi$$

with

$$\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab} + X^{ab}$$

- Standard equations
- Particular case: Standard Model Extension
- More general cases with charge non–conservation $\dot{Q} \neq 0 \leftrightarrow \dot{\alpha} \neq 0$

Test theories, Phenomenology

Phenomenology = Generalizations of Maxwell and Dirac equations

$$\begin{aligned} 4\pi j^\mu &= \eta^{\mu\rho}\eta^{\nu\sigma}\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\nu\sigma}\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\sigma} F_{\rho\sigma} \\ &\qquad + \chi^{\mu\rho\nu\sigma\tau}\partial_\nu\partial_\tau F_{\rho\sigma} + \ldots \\ 0 &= i\gamma^a D_a\psi + m\psi + M\psi + \gamma^{ab} D_a D_b \psi + \ldots \end{aligned}$$

with

$$\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab} + X^{ab}$$

- Standard equations
- Particular case: Standard Model Extension
- More general cases with charge non–conservation $\dot{Q} \neq 0 \leftrightarrow \dot{\alpha} \neq 0$
- Higher derivative models

Test theories, Phenomenology

Phenomenology = Generalizations of Maxwell and Dirac equations

$$
\begin{aligned}
4\pi j^\mu &= \eta^{\mu\rho}\eta^{\nu\sigma}\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\nu\sigma}\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\sigma} F_{\rho\sigma} \\
&\qquad + \chi^{\mu\rho\nu\sigma\tau}\partial_\nu\partial_\tau F_{\rho\sigma} + \ldots + \zeta^{\mu\rho\sigma\tau\nu} F_{\rho\sigma} F_{\tau\nu} \\
0 &= i\gamma^a D_a\psi + m\psi + M\psi + \gamma^{ab} D_a D_b \psi + \ldots + N(\psi)\psi
\end{aligned}
$$

with

$$
\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab} + X^{ab}
$$

- Standard equations
- Particular case: Standard Model Extension
- More general cases with charge non–conservation $\dot{Q} \neq 0 \leftrightarrow \dot{\alpha} \neq 0$
- Higher derivative models
- Non–linear terms

Test theories, Phenomenology

Phenomenology = Generalizations of Maxwell and Dirac equations

$$
\begin{aligned}
4\pi j^\mu &= \eta^{\mu\rho}\eta^{\nu\sigma}\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\nu\sigma}\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\sigma} F_{\rho\sigma} \\
&\qquad + \chi^{\mu\rho\nu\sigma\tau}\partial_\nu\partial_\tau F_{\rho\sigma} + \ldots + \zeta^{\mu\rho\sigma\tau\nu} F_{\rho\sigma} F_{\tau\nu} \\
0 &= i\gamma^a D_a\psi + m\psi + M\psi + \gamma^{ab} D_a D_b \psi + \ldots + N(\psi)\psi
\end{aligned}
$$

with

$$
\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab} + X^{ab}
$$

- Standard equations
- Particular case: Standard Model Extension
- More general cases with charge non–conservation $\dot{Q} \neq 0 \leftrightarrow \dot{\alpha} \neq 0$
- Higher derivative models
- Non–linear terms
- One may even add nonlocal Finslerian terms (discussed for QG, VSR, ...)

Test theories, Phenomenology

Phenomenology = Generalizations of Maxwell and Dirac equations

$$\begin{aligned} 4\pi j^\mu &= \eta^{\mu\rho}\eta^{\nu\sigma}\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\nu\sigma}\partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\sigma} F_{\rho\sigma} \\ &\qquad + \chi^{\mu\rho\nu\sigma\tau}\partial_\nu\partial_\tau F_{\rho\sigma} + \ldots + \zeta^{\mu\rho\sigma\tau\nu} F_{\rho\sigma} F_{\tau\nu} \\ 0 &= i\gamma^a D_a\psi + m\psi + M\psi + \gamma^{ab} D_a D_b \psi + \ldots + N(\psi)\psi \end{aligned}$$

with

$$\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab} + X^{ab}$$

- Standard equations
- Particular case: Standard Model Extension
- More general cases with charge non–conservation $\dot{Q} \neq 0 \leftrightarrow \dot{\alpha} \neq 0$
- Higher derivative models
- Non–linear terms
- One may even add nonlocal Finslerian terms (discussed for QG, VSR, ...)

Even if QG does not give all terms, one learns about the structure of QG

Outline

Schiff's conjecture

UFF implies the EEP, or: Violation of LLI or LPI implies violation of UFF

UFF is a universal tool to look for violations of standard pyhsics

In reply to the Schiff conjecture: Ni, PRL 1977

Theorem: In a neutral system which Lagrangian densitiy is given by

$$\mathscr{L} = -\frac{1}{16\pi}\Lambda^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - A_\mu j^\mu \sqrt{-g} - \sum_i m_i \frac{ds_i}{dt}\delta(x - x_i)$$

the UFF holds if and oly if

$$\Lambda^{\mu\nu\rho\sigma} = \sqrt{-g}\left(\frac{1}{2}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) + \phi\epsilon^{\mu\nu\rho\sigma}\right),$$

ϕ pseudoscalar field (axion)

Only small loophole $\Rightarrow$ UFF still is most important principle

Outline

Outline

UFF and charge

Standard theory

- In standard theory from ordinary coupling (deWitt & Brehme, AP 1968) $a^\mu = \alpha \lambda_C R^\mu{}_\nu v^\nu \sim 10^{-35}\ \mathrm{m/s}^2$

Anomalous coupling

- Anomalous coupling (Dittus, C.L., Selig, GRG 2004)

$$H = \frac{\boldsymbol{p}^2}{2m} + mU(\boldsymbol{x}) + \kappa e U(\boldsymbol{x}) = \frac{\boldsymbol{p}^2}{2m} + m\left(1 + \kappa \frac{e}{m}\right) U(\boldsymbol{x})\,.$$

- Charge dependent anomalous gravitational mass
- Can be generalized to charge dep. anom. inertial mass (e.g. Rohrlich 2000)

$\Rightarrow$ Charge dependent Eötvös factor

- It is possible to choose κ's such that for neutral composite matter UFF is fulfilled while for isolated charges UFF is violated
- No underlying theory known

UFF and spin

Standard theory

- In standard theory from ordinary coupling: $a^\mu = \lambda_C R^\mu{}_{\nu\rho\sigma} v^\nu S^{\rho\sigma} \Rightarrow$ violation of UFF at the order $10^{-20}\ \mathrm{m/s^2}$, beyond experiment

Anomalous coupling

- Speculations: violation P, C, and T symmetry in gravitational fields (Leitner & Okubo 1964, Moody & Wilczek 1974) suggest

$$V(r) = U(r)\left[1 + A_1(\boldsymbol{\sigma}_1 \pm \boldsymbol{\sigma}_2)\cdot\widehat{\boldsymbol{r}} + A_2(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)\cdot\widehat{\boldsymbol{r}}\right]$$

- One body (e.g., the Earth) is unpolarized $\rightarrow$

$$V(r) = U(r)\left(1 + A\boldsymbol{\sigma}\cdot\widehat{\boldsymbol{r}}\right)$$

  Hyperfine splittings of H ground state: $A_p \leq 10^{-11}$, $A_e \leq 10^{-7}$

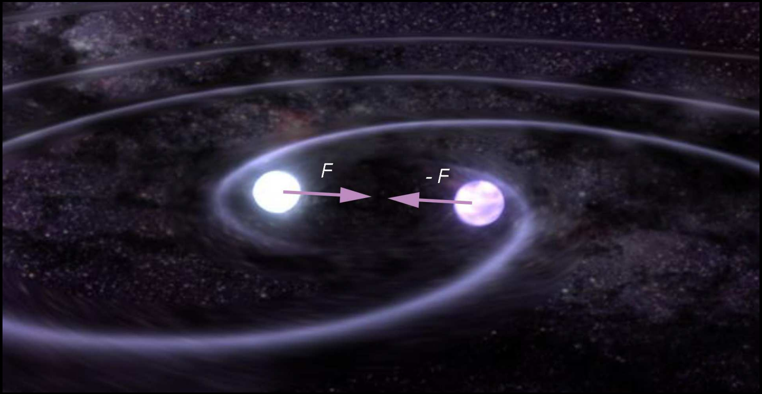
- Hari Dass 1976, 1977, includes velocity of the particles

$$V(r) = U_0(r)\left[1 + A_1\boldsymbol{\sigma}\cdot\widehat{\boldsymbol{r}} + A_2\boldsymbol{\sigma}\cdot\frac{\boldsymbol{v}}{c} + A_3\widehat{\boldsymbol{r}}\cdot\left(\boldsymbol{\sigma}\times\frac{\boldsymbol{v}}{c}\right)\right]$$

Outline

Creation of the gravitational field

If masses of different composition react differently on a given gravitational field, then it is natural that masses of different composition also create a different gravitational field
$\Rightarrow$ question of equality of active and passive gravitational masses



Active and passive mass

Gravitationally bound two–body system (Bondi, RMP 1957)

$$m_{1\mathrm{i}}\ddot{\boldsymbol{x}}_1 = m_{1\mathrm{p}}m_{2\mathrm{a}}\frac{\boldsymbol{x}_2-\boldsymbol{x}_1}{|\boldsymbol{x}_2-\boldsymbol{x}_1|^3}$$

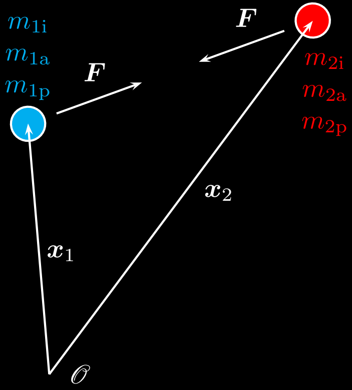
$$m_{2\mathrm{i}}\ddot{\boldsymbol{x}}_2 = m_{2\mathrm{p}}m_{1\mathrm{a}}\frac{\boldsymbol{x}_1-\boldsymbol{x}_2}{|\boldsymbol{x}_1-\boldsymbol{x}_2|^3}$$

center–of–mass and relative coordinate

$$\boldsymbol{X} := \frac{m_{1\mathrm{i}}}{M_\mathrm{i}}\boldsymbol{x}_1 + \frac{m_{2\mathrm{i}}}{M_\mathrm{i}}\boldsymbol{x}_2$$

$$\boldsymbol{x} := \boldsymbol{x}_2 - \boldsymbol{x}_1$$

$M_\mathrm{i} = m_{1\mathrm{i}} + m_{2\mathrm{i}}$ = total inertial mass. Then



Active and passive mass

Decoupled dynamics of relative coordinate

$$\ddot{\boldsymbol{X}} = \frac{m_{1\mathrm{p}} m_{2\mathrm{p}}}{M_\mathrm{i}} C_{21} \frac{\boldsymbol{x}}{|\boldsymbol{x}|^3} \qquad \text{with} \qquad C_{21} = \frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}} - \frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}}$$

$$\ddot{\boldsymbol{x}} = -\frac{m_{1\mathrm{p}} m_{2\mathrm{p}}}{m_{1\mathrm{i}} m_{2\mathrm{i}}} \left(m_{1\mathrm{i}} \frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}} + m_{2\mathrm{i}} \frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}} \right) \frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}$$

- $C_{21} = 0$: ratio of the active and passive masses are equal for both particles
- $C_{21} \neq 0$: $\Rightarrow$ self–acceleration of center of mass

Interpretation

$\ddot{\boldsymbol{X}} \neq 0 \quad \Leftrightarrow \quad C_{12} \neq 0 \quad \Leftrightarrow$

- Violation of law of reciprocal action or of *actio* $=$ *reactio* for gravity
- The gravitational field created by masses of same weight depends on its composition. Has the same status as the UFF.

Requires experimental tests ...

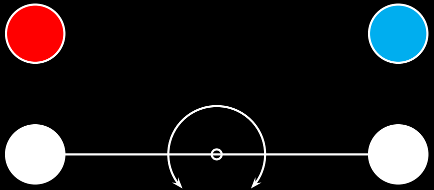
Experiment testing $m_{\mathrm{ga}} = m_{\mathrm{gp}}$

Measurement of relative acceleration

Step 1: Take two masses with $m_{\mathrm{p1}} = m_{\mathrm{p2}}$ (equal weight)

Step 2: Test active equality of these two masses with torsion balance

Experimental setup: Torsion balance with equal passive masses reacting on m_{a1} and m_{a2}



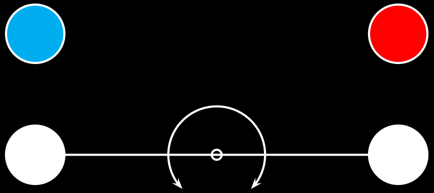
No effect has been seen: $C_{12} \leq 5 \cdot 10^{-5}$ (Kreuzer, PR 1868)

Experiment testing $m_{\rm ga} = m_{\rm gp}$

Measurement of relative acceleration

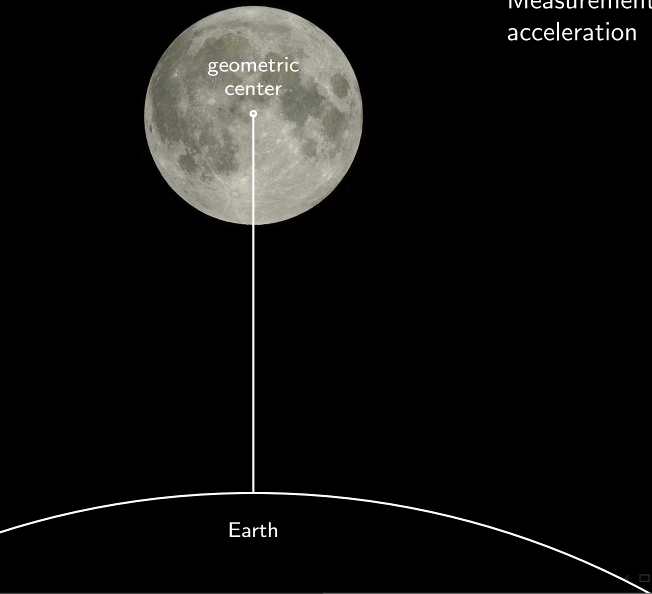
- Step 1: Take two masses with $m_{\rm p1} = m_{\rm p2}$ (equal weight)
- Step 2: Test active equality of these two masses with torsion balance

Experimental setup: Torsion balance with equal passive masses reacting on $m_{\rm a1}$ and $m_{\rm a2}$



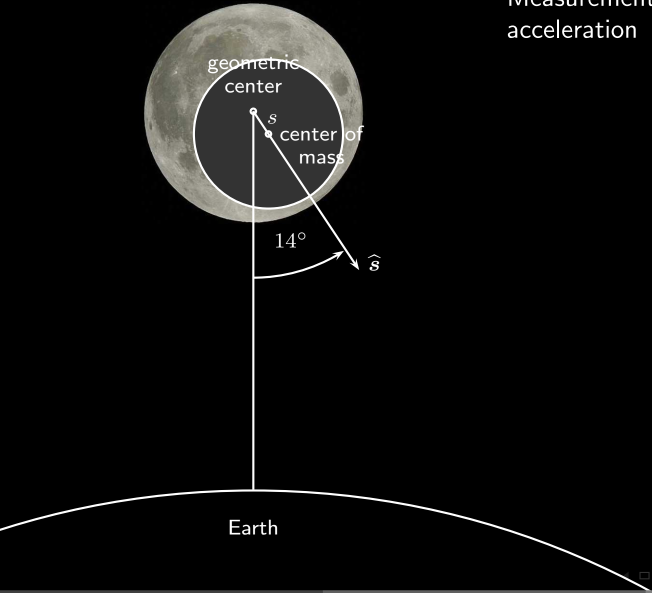
No effect has been seen: $C_{12} \leq 5 \cdot 10^{-5}$ (Kreuzer, PR 1868)

Experiment testing $m_{\mathrm{ga}} = m_{\mathrm{gp}}$



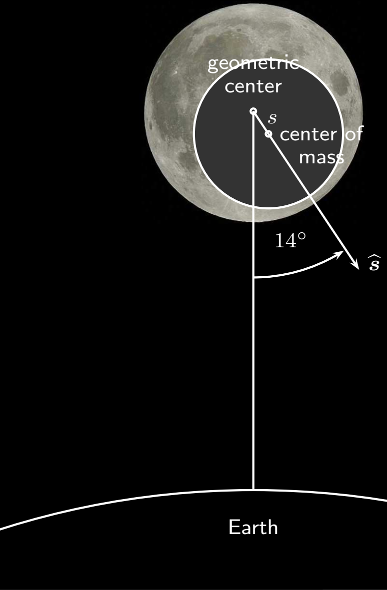
Measurement of center–of–mass acceleration

Experiment testing $m_{\mathrm{ga}} = m_{\mathrm{gp}}$



Measurement of center–of–mass acceleration

Experiment testing $m_{\mathrm{ga}} = m_{\mathrm{gp}}$



Measurement of center–of–mass acceleration

$$\frac{\boldsymbol{F}_{\mathrm{self}}}{F_{\mathrm{EM}}} = C_{\mathrm{Al-Fe}} \frac{M_{\mathrm{M}}}{M_{\oplus}} \frac{r_{\mathrm{EM}}^2}{r_{\mathrm{M}}^2} \frac{s}{r_{\mathrm{M}}} \frac{\rho}{\Delta\rho} \hat{\boldsymbol{s}}$$

Effect of tangential part: increase of orbital angular velocity

$$\frac{\Delta\omega}{\omega} = 6\pi \frac{F_{\mathrm{self}}}{F_{\mathrm{EM}}} \sin 14^\circ \text{ per month}$$

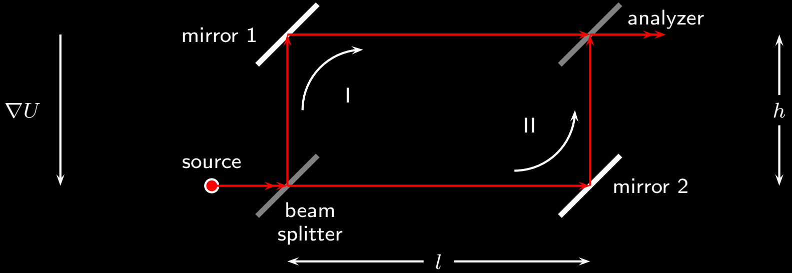
From LLR $\frac{\Delta\omega}{\omega} \leq 10^{-12}$ per month

$$\Rightarrow \qquad C_{\mathrm{Al-Fe}} \leq 7 \cdot 10^{-13}$$

Bartlett & van Buren, PRL 1986
significant improvement with new LLR data and moon orbiter data possible

Outline

The general phase shift



wave functions at analyzer

$$\varphi_{\mathrm{I}}(x) = \exp\left(-\frac{1}{2}\oint_{x_0}^{x}\theta ds\right)\exp\left(-i\oint_{x_0}^{x}p_\mu dx^\mu\right)a_0$$

$$\varphi_{\mathrm{II}}(x) = \exp\left(-\frac{1}{2}\oint_{x_0}^{x}\theta ds\right)\exp\left(-i\oint_{x_0}^{x}p_\mu dx^\mu\right)a_0$$

The general phase shift

The intensity

Intensity of the two interfering wave functions at one port of the analyzer

$$I = |\varphi_{\mathrm{I}} + \varphi_{\mathrm{II}}|^2 = 2\,(1 + \cos\Delta\phi)\,|a_0|^2\,,$$

with phase shift

$$\boxed{\Delta\phi = \oint p}$$

integration along classical trajectory

quasiclassical limit

$$const. = E = \frac{p^2}{2m} + mgh$$

then one obtains phase shift (Colella & Overhauser, PRL 1974)

$$\delta\phi = \frac{mglh}{\hbar v}$$

depends on mass $\longrightarrow$ Equivalence Principle?

On the Equivalence Principle

Model

Schrödinger equation in gravitational field

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\phi + m\ U\psi$$

Phase shift

For pure gravitational acceleration

- atom interferom. (Bordé 1989)

$$\boxed{\delta\phi = \quad \boldsymbol{k}\cdot\boldsymbol{g}\ T^2}$$

- neutron interf. (CL, GRG 1996)

$$\delta\phi = \boldsymbol{C}\cdot\boldsymbol{g}\ T^2$$

Discussion

- Exact quantum result
- UFF exactly fulfilled
- Does not depend on $\hbar$
- $\hbar$ comes in by introducing classical notions
  - height $= h = v_z T = \frac{\hbar k}{m} T$
  - length $= l = v_0 T$

  $$\delta\phi = k_z g T^2 = \frac{mghl}{\hbar v_0}$$
- $\delta\phi = k_z g T^2$ contains experimentally given quantities only
- Though quantum system is nonlocal, measurement of the quantum phase via interference yields test of UFF

On the Equivalence Principle

Model

Schrödinger equation in gravitational field

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_\mathrm{i}}\Delta\phi + m_\mathrm{g}U\psi$$

Phase shift

For pure gravitational acceleration

- atom interferom. (Bordé 1989)

$$\boxed{\delta\phi = \frac{m_\mathrm{g}}{m_\mathrm{i}}\,\boldsymbol{k}\cdot\boldsymbol{g}\,T^2}$$

- neutron interf. (CL, GRG 1996)

$$\delta\phi = \boldsymbol{C}\cdot\boldsymbol{g}\,T^2$$

Discussion

- Exact quantum result
- UFF exactly fulfilled
- Does not depend on $\hbar$
- $\hbar$ comes in by introducing classical notions
  - height $= h = v_z T = \frac{\hbar k}{m} T$
  - length $= l = v_0 T$

$$\delta\phi = k_z g T^2 = \frac{mghl}{\hbar v_0}$$

- $\delta\phi = k_z g T^2$ contains experimentally given quantities only
- Though quantum system is nonlocal, measurement of the quantum phase via interference yields test of UFF

Space–time fluctuations

The model

- General expectation for Quantum Gravity: space–time fluctuates
- Simplest model of space–time fluctuations $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Simplest matter system: Klein–Gordon in this fluctuating space-time

- Relativistic approximation + spatial averaging

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\left(\delta^{ij} + \tilde{\alpha}^{ij} + \gamma^{ij}(t)\right)\partial_i\partial_j\psi - mU\psi$$

$\widetilde{\alpha}^{ij} \leftrightarrow$ spectral noise density of fluctuations

- Particular model: $m_{\mathrm{i}} = \dfrac{m}{1+\widetilde{\alpha}}, \quad \widetilde{\alpha} \sim \left(\dfrac{l_{\mathrm{Planck}}}{l_{\mathrm{system}}}\right)^\beta$

Result

$\Rightarrow$ anomalous inertial mass $\rightarrow$ apparent violation of UFF

Example: Cs and H: $\eta_{\beta=1} = 10^{-20}$, $\eta_{\beta=2/3} = 10^{-15}$ (holographic noise)
Also decoherence, modif. wave packet spreading (Göklü & C.L. CQG 2008, 2009)

Outline

Summary

- Today's understanding of physics is not complete
- This requires "new physics" – quantum gravity
- Most prominent scenario is string theory, others are Loop Quantum Gravity, renormalization group ansatz, dynamical triangulization, ...
- The most promising sector to find "new physics" is the Equivalence Principle

Therefore:

There is a highest priority scientific need for MICROSCOPE

Summary

- Today's understanding of physics is not complete
- This requires "new physics" – quantum gravity
- Most prominent scenario is string theory, others are Loop Quantum Gravity, renormalization group ansatz, dynamical triangulization, ...
- The most promising sector to find "new physics" is the Equivalence Principle

Therefore:

There is a highest priority scientific need for MICROSCOPE

MICROSCOPE + ACES + STE QUEST: Complete space–based laboratory program to investigate the fundamentals of the gravity sector